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Decision Theory

Final Exam

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Each of the following questions has four possible answers, of which only one is correct. If you solve a problem correctly, you will receive one point; if you do not answer a problem you will receive zero points; and if you answer a problem incorrectly, you will lose one half point. It is therefore better to not answer a question at all, than to answer the question incorrectly; better still is to answer the question correctly.

The exam consists of this title page and seven further pages with problems: please count the pages. Behind the pages with problems there are five empty pages for your calculations. These pages may not be separated from the other pages.

Admitted Aid: Non-programmable calculator.

Be careful in marking your answers – if more than one answer to a problem is marked, the problem will be valued as incorrect.

- A car dealership sells 300 cars a year. To keep a car in storage costs \$192 a year. The cost of ordering new cars from the manufacturer involves a fixed cost of \$200 plus \$80 per car. Assuming that cars are sold at a constant rate, how many times per year should the car dealer order cars to minimize his annual total costs?

- 25
- 15
- 12
- 10

- A company has two factories, each manufacturing the same product. Factory A produces x units of the product at the cost of $2x^2 + 50000$ dollars, and factory B can produce y units at a cost of $y^2 + 40000$. If an order for 1200 units is to be filled, how should the production be distributed among the two factories in order to minimize the total cost of production?

- A 0 units and B 1200 units.
- A 400 units and B 800 units.
- A 600 units and B 600 units.
- A 800 units and B 400 units.

- You are decorating a room for your indecisive grandmother, who will not tell you what color paint to use, but will give you a strict and consistent preference between any pair of colors. The choice of paint is restricted to seven colors. You have asked her seven questions and learnt that she holds

Beige \succ Pink, Beige \succ Green, Pink \succ White, Blue \succ Beige,
Green \succ White, Brown \succ Lilac, Lilac \succ Beige.

What is the next and last question you must ask her in order to determine her choice of color for the room?

- Do you prefer Brown or Blue?
- Do you prefer Blue or Lilac?
- Do you prefer Pink or Green?
- Do you prefer Blue or White?

- An individual has the choice between receiving a sure payment of \$90 and investing \$100 in order to play one of two gambles. In the first gamble he can win \$500 with a probability of 1/2 or \$100 with a probability of 1/2. In the second gamble he has a 50-50 chance of winning \$325 or \$136. Assuming that the individual's utility over wealth w is given by $u(w) = \sqrt{w}$, what should he choose?

- Indifferent between Gamble 1 and 2.
- Gamble 2.
- Gamble 1.
- The sure payment.

- Xenia, Yashi, and Zelda are contestants in a game show where each has a 1% chance of winning a prize of \$100 000 or otherwise leaving with only \$100. The contestants' utility functions of wealth are given by

Xenia: $u_x(w) = \frac{1}{2}w^2$

Yashi: $u_y(w) = \sqrt{w}$

Zelda: $u_z(w) = 2w$

The show master offers each of them \$1500 if they quit the show. Who will accept the offer?

- No one.
- Only Yashi.
- Yashi and Zelda.
- All three.

- A shop owner has a store worth DM 750 000. She assumes that there is a 2% chance of the store being destroyed by fire. The shop owner has the possibility of buying DM 1 amount of coverage for DM 0.02. How much coverage will the shop owner buy if she is risk averse with a utility function over wealth $u(w) = \sqrt{w}$?

- DM 1 000 000
- DM 750 000
- DM 500 000
- DM 15 000

- Consider the following four lotteries:

$$A = \{(100, 0.25), (110, 0.5), (120, 0.25)\}$$

$$B = \{(90, 0.25), (100, 0.5), (120, 0.25)\}$$

$$C = \{(100, 0.25), (110, 0.5), (100, 0.25)\}$$

$$D = \{(90, 0.25), (120, 0.5), (110, 0.25)\}$$

A decision maker is known to hold the following indifferences with the reference lotteries

$$100 \sim \{(120, 0.5), (90, 0.5)\}$$

$$110 \sim \{(120, 0.8), (90, 0.2)\}$$

With this information, which of the four lotteries should the decision maker choose in order to maximize expected utility?

- D
- C
- B
- A

- A team of scientists is due to spend six months in Antarctica carrying out research. One major piece of equipment they will be taking is subject to breakdowns caused by the sudden failure of a particular component. Because a failed component cannot be repaired, the team intends to carry a stock of spare units of the component, but it will cost them roughly \$3000 for each spare unit they take with them. However, if the equipment breaks down and a spare is not available, a new unit will have to be specially flown in, and the team will incur a total cost of \$4000 for each unit that is delivered in this way. An engineer who will be travelling with the team is certain that the number of spares required will not exceed three.

How many spares should the team carry with them if their objective is to minimize costs and they decide according to the Hurwicz rule with $\lambda = 1/2$?

- 0
- 1
- 2
- 3

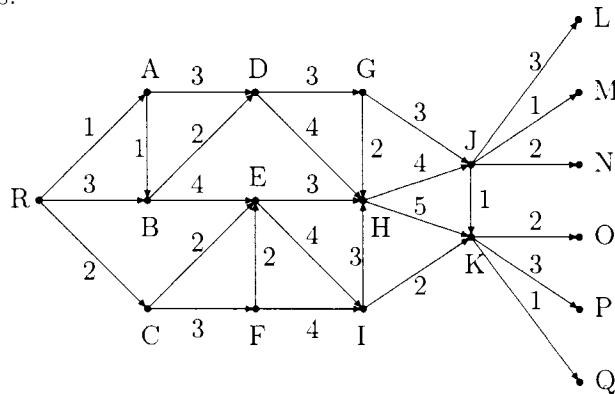
- Consider the decision of the team of scientists in the previous problem. After further research the engineer has estimated that the number of spares that will be required during the six months follows the following probability distribution:

| | | | | |
|-----------------|-----|-----|-----|-----|
| Spares required | 0 | 1 | 2 | 3 |
| Probability | 0.2 | 0.3 | 0.4 | 0.1 |

Determine the number of spares that the team should carry with them, if their objective is to minimize expected costs.

- 0
- 1
- 2
- 3

- Consider the sequential decision process given by the following graph. The decision process begins with the decision at R and ends at one of the outcomes L, M, N, O, P, or Q. The numbers along the edges represent the costs of taking the corresponding decisions.



The cost minimizing sequence of choices yields an aggregate cost of

- 13
- 12
- 11
- 10

- Uncle Euclid, an ancient Athenian entrepreneur, is anxious to start a new business. Three possibilities are available to him, and the success of each enterprise will depend on the volatile tastes of the consumers of Athens. In analyzing his decision problem, Euclid has classified the market conditions into three possible states, $\theta_1, \theta_2, \theta_3$, and he assesses his profit (in monetary units) for each of the three enterprises, a_1, a_2, a_3 , as:

| | θ_1 | θ_2 | θ_3 |
|-------|------------|------------|------------|
| a_1 | 5 | 0 | 13 |
| a_2 | 6 | 7 | 7 |
| a_3 | 2 | 4 | 9 |

To help him determine the future state of the market, Euclid may, at a price, consult the Delphi Oracle, who will tell him for certain which of the states will prevail. Calculate the maximum that Euclid would be prepared to pay the Oracle for this information if he makes his decision according to the minimal regret rule:

- 4
 - 6
 - 5
 - 3
- Imagine that you are the candidate in a game show with five rounds of statements, to which you must respond only 'true' or 'false'. If your response is incorrect, you end the game with nothing. However, if your response is correct, you can leave with \$5 000 after the first round, \$15 000 after the second, \$40 000 after the third, \$75 000 after the fourth, or \$100 000 if you respond correctly to the fifth statement. After each correct response you must choose whether you wish to take the money or invest it in the next round. Since you do not know the next statement beforehand, you consider guessing in the next round, so that your chance of a correct response is 50%. Assuming that you are risk neutral, after which statement should you leave the game?
- After the 5th.
 - After the 4th.
 - After the 3rd.
 - After the 2nd.

- A mining company is carrying out a survey in a region of Western Australia. On the basis of preliminary results, the company's senior geologist estimates that there is a 50% probability that a particular mineral will be found in quantities that would justify commercial investment in the region. Further research is then carried out and this suggests that commercially viable quantities of the mineral will be found. It is estimated that this research gives a correct indication in 2 out of 3 cases. Revise the senior geologist's estimate of finding the mineral in the light of the research results.

3/4.

1/2.

1/3.

2/3.

- Estimates show that 0.3% of the U.S. population is carrying the sexually transmitted HIV virus, which is known to cause the deadly disease AIDS. In order to study the spread of the HIV virus in the population, it was suggested that the U.S. Congress pass a law requiring that couples applying for marriage should take the blood test for the HIV virus. The HIV blood test is considered very "effective" since:

– A person with the HIV virus has a 95% chance to test positive, and

– An HIV free person has a 4% chance to test positive.

In order to check the effectiveness of the test, how high is the probability that a person taking the HIV virus test has the disease, given that the test is positive?

2.85%.

6.67%.

42.35%.

95%.

- The owner of a small business is unhappy with the service she has been receiving from her bank and has decided to move her account to a rival bank. Her decision on which bank to choose will be based not only on the estimated annual bank *charges* which each bank will levy, but also on the following benefit attributes, for which she has also assessed the swing weights (bold numbers in parentheses):
 - the *proximity* of the local branch (**20**),
 - whether the local branch has a small business *adviser* (**50**),
 - the maximum automatic *loan* allowed (**100**),
 - whether a *telephone* banking facility is offered (**80**).

The alternative banks are listed below, together with their estimated annual costs and the scores the business owner has allocated for each of the benefit attributes.

| <i>Bank</i> | <i>Charges</i> | <i>Proximity</i> | <i>Adviser</i> | <i>Loan</i> | <i>Telephone</i> |
|-------------|----------------|------------------|----------------|-------------|------------------|
| Central | 3000 | 0 | 100 | 40 | 0 |
| Northern | 5000 | 100 | 100 | 80 | 0 |
| Direct | 2000 | 70 | 0 | 100 | 100 |
| Royal | 1000 | 30 | 0 | 0 | 100 |
| Marks | 4000 | 90 | 100 | 20 | 0 |

Contrasting annual Charges with the overall benefits of each bank, which are the efficient choices?

- Direct and Royal.
 - Northern and Royal.
 - Northern and Direct.
 - Northern and Marks.
- Consider a two-person strategic form game such that $S_1 = S_2 = \mathbb{R}$. The utility functions of the two players are

$$u_1(x, y) = xy^2 - x^2 \quad \text{and} \quad u_2(x, y) = 8y - xy^2.$$

The Nash-equilibrium strategies of the game are

- $x = 2, y = 3$.
- $x = 2, y = 2$.
- $x = 2, y = 1$.
- $x = 1, y = 2$.