

Answer-Sheet – Group A
Exam: Decision Theory SS 2003 [5072]

Please mark your solutions in the following tables. You must fill out the **whole** circle. Do not use a pencil, but a blue or black pen, otherwise a correct marking might not be possible.

Do not forget to sign the sheet at the bottom!

Good Luck

Family Name: First Name:

Matr.-No.: Faculty:

Matriculation Number

| | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|----------|-----------------------|-----------------------|-----------------------|-----------------------|-----------------------|-----------------------|-----------------------|-----------------------|-----------------------|-----------------------|
| 1. digit | <input type="radio"/> | <input type="radio"/> | <input type="radio"/> | <input type="radio"/> | <input type="radio"/> | <input type="radio"/> | <input type="radio"/> | <input type="radio"/> | <input type="radio"/> | <input type="radio"/> |
| 2. digit | <input type="radio"/> | <input type="radio"/> | <input type="radio"/> | <input type="radio"/> | <input type="radio"/> | <input type="radio"/> | <input type="radio"/> | <input type="radio"/> | <input type="radio"/> | <input type="radio"/> |
| 3. digit | <input type="radio"/> | <input type="radio"/> | <input type="radio"/> | <input type="radio"/> | <input type="radio"/> | <input type="radio"/> | <input type="radio"/> | <input type="radio"/> | <input type="radio"/> | <input type="radio"/> |
| 4. digit | <input type="radio"/> | <input type="radio"/> | <input type="radio"/> | <input type="radio"/> | <input type="radio"/> | <input type="radio"/> | <input type="radio"/> | <input type="radio"/> | <input type="radio"/> | <input type="radio"/> |
| 5. digit | <input type="radio"/> | <input type="radio"/> | <input type="radio"/> | <input type="radio"/> | <input type="radio"/> | <input type="radio"/> | <input type="radio"/> | <input type="radio"/> | <input type="radio"/> | <input type="radio"/> |
| 6. digit | <input type="radio"/> | <input type="radio"/> | <input type="radio"/> | <input type="radio"/> | <input type="radio"/> | <input type="radio"/> | <input type="radio"/> | <input type="radio"/> | <input type="radio"/> | <input type="radio"/> |

Solutions

| | A | B | C | D |
|---|-----------------------|-----------------------|-----------------------|-----------------------|
| 1 | <input type="radio"/> | <input type="radio"/> | <input type="radio"/> | <input type="radio"/> |
| 2 | <input type="radio"/> | <input type="radio"/> | <input type="radio"/> | <input type="radio"/> |
| 3 | <input type="radio"/> | <input type="radio"/> | <input type="radio"/> | <input type="radio"/> |
| 4 | <input type="radio"/> | <input type="radio"/> | <input type="radio"/> | <input type="radio"/> |
| 5 | <input type="radio"/> | <input type="radio"/> | <input type="radio"/> | <input type="radio"/> |
| 6 | <input type="radio"/> | <input type="radio"/> | <input type="radio"/> | <input type="radio"/> |
| 7 | <input type="radio"/> | <input type="radio"/> | <input type="radio"/> | <input type="radio"/> |
| 8 | <input type="radio"/> | <input type="radio"/> | <input type="radio"/> | <input type="radio"/> |

| | A | B | C | D |
|----|-----------------------|-----------------------|-----------------------|-----------------------|
| 9 | <input type="radio"/> | <input type="radio"/> | <input type="radio"/> | <input type="radio"/> |
| 10 | <input type="radio"/> | <input type="radio"/> | <input type="radio"/> | <input type="radio"/> |
| 11 | <input type="radio"/> | <input type="radio"/> | <input type="radio"/> | <input type="radio"/> |
| 12 | <input type="radio"/> | <input type="radio"/> | <input type="radio"/> | <input type="radio"/> |
| 13 | <input type="radio"/> | <input type="radio"/> | <input type="radio"/> | <input type="radio"/> |
| 14 | <input type="radio"/> | <input type="radio"/> | <input type="radio"/> | <input type="radio"/> |
| 15 | <input type="radio"/> | <input type="radio"/> | <input type="radio"/> | <input type="radio"/> |
| 16 | <input type="radio"/> | <input type="radio"/> | <input type="radio"/> | <input type="radio"/> |

Student Signature: _____

Prof. Dr. Matthias Raith
Department of Economics
Otto-von-Guericke-University Magdeburg

Decision Theory
(5072)

Final Exam

July 15, 2003

GROUP A

Each of the following questions has four possible answers, of which only one is correct. If you solve a problem correctly, you will receive one point; if you do not answer a problem you will receive zero points; and if you answer a problem incorrectly, you will lose one half point. It is therefore better to not answer a question at all, than to answer the question incorrectly; better still is to answer the question correctly.

The exam consists of this title page and 16 problems; please count the pages. The pages may not be separated.

You will receive a separate form for your answers. Please make sure that the GROUP indicated on the answer form is the same as on your set of problems. Before you begin the exam, please fill in your personal data on the answer form. Be careful in marking your answers on the answer form; if more than one answer to a problem is marked, the answer will be valued as incorrect. If you have a correction to make, please mark this clearly.

Admitted Aids: Non-programmable calculator, dictionary.

1. A car dealership sells 300 cars a year. To keep a car in storage costs \$192 a year. The cost of ordering new cars from the manufacturer involves a fixed cost of \$200 plus \$80 per car. Assuming that cars are sold at a constant rate, how many times per year should the car dealer order cars to minimize his annual total costs?
- (A) 10
 - (B) 12
 - (C) 15
 - (D) 25

2. You are attending a large school reunion, where everyone only seems to be talking about how much money they are making or how many children they have. Your former school director has suggested ordering all schoolmates according to the relation “**earns more money or has more children than**”. In order to reveal your success as a decision analyst you comment that this relation is
- (A) both complete and transitive.
 - (B) complete but not transitive.
 - (C) transitive but not complete.
 - (D) neither complete nor transitive.

3. A company has two factories, each manufacturing the same product. Factory A produces x units of the product at the cost of $2x^2 + 50000$ dollars, and factory B can produce y units at a cost of $y^2 + 40000$. If an order for 1200 units is to be filled, how should the production be distributed among the two factories in order to minimize the total cost of production?
- (A) A 800 units and B 400 units.
 - (B) A 600 units and B 600 units.
 - (C) A 400 units and B 800 units.
 - (D) A 0 units and B 1200 units.

4. A chemical company is expanding its operations and a disused woolen mill is to be converted into a processing plant. Four companies have submitted designs for the equipment which will be installed in the mill, and a choice has to be made between them. The manager of the chemical company has identified three attributes which he considers to be important in the discussion: 'cost,' 'environmental impact,' and 'reliability.' He has assessed how well each design performs on each attribute by allocating values on a scale from 0 (worst) to 100 (best). These values are shown below, together with the costs which will be incurred if a design is chosen:

| Design | Cost (\$) | Env. Impact | Reliability |
|---------------|------------------|--------------------|--------------------|
| A | 90 000 | 20 | 100 |
| B | 110 000 | 70 | 0 |
| C | 170 000 | 100 | 90 |
| D | 60 000 | 0 | 50 |

Which design should be chosen if the manager thinks that points for reliability are twice as valuable as points for the benefit of environmental impact, and if he is willing to pay \$1000 for each extra point of reliability?

- (A) A
- (B) B
- (C) C
- (D) D

5. Vincent is looking for a job. Next to his monthly salary (in Euros) he is also interested in other benefits such as job flexibility, the skills he will be learning, the number of vacation days, benefits in health, dental, and retirement support, and last but not least the enjoyment of the job. After identifying five potential jobs, which he could rank according to salary, Vincent also achieved an assessment of the jobs according to the other attributes using the SMART method, and in order to determine the importance of the non-salary attributes, he derived the swing weights of these attributes. His results are given in the following table:

| Job | Salary | Flexibility | Skills | Vacation | Benefits | Enjoyment |
|----------------|--------|-------------|--------|----------|----------|-----------|
| A | 2400 | 40 | 100 | 50 | 75 | 60 |
| B | 2200 | 0 | 75 | 50 | 75 | 0 |
| C | 2000 | 75 | 25 | 90 | 100 | 100 |
| D | 1900 | 75 | 0 | 100 | 25 | 100 |
| E | 1800 | 100 | 50 | 0 | 25 | 60 |
| Swing Weights: | | 80 | 100 | 60 | 40 | 60 |

Considering the total benefits (measured on a scale of 0 to 100) and the costs of each job, Vincent generally believes that 6 benefit points are worth € 100. With this information, which job should Vincent choose?

- (A) A
- (B) C
- (C) D
- (D) E

6. Consider a decision maker whose preferences are characterized by a linear utility function. A lottery offers her the opportunity to win \$800 with a probability of p and \$200 with a probability of $1 - p$. *How large is the probability p if the decision maker's certainty equivalent is \$400?*

(A) $p = 3/4$

(B) $p = 2/3$

(C) $p = 1/2$

(D) $p = 1/3$

7. Producer A values a lottery yielding \$10 000 with a probability of 20% and \$1 000 with a probability of 80% the same as a sure payment of \$3 000. Producer B, on the other hand, is indifferent between a sure payment of \$7 000 and a lottery giving \$10 000 with a probability of 70% and \$1 000 with a probability of 30%. Concerning the risk attitudes of the two producers, one can say that
- (A) Producer A is risk-seeking and producer B risk-averse.
 - (B) Producer A is risk-averse and producer B risk-seeking.
 - (C) Producer A is risk-neutral and producer B risk-averse.
 - (D) Producer A is risk-seeking and producer B risk-neutral.

8. A risk-averse decision maker with a monotonically increasing von-Neumann-Morgenstern utility function $u(x)$ claims that she values the lottery

$$L = \{(20; 0.3), (5; 0.7)\}$$

just as much as the sure payment of 8. If $u(20) = 250$ and $u(5) = 0$, which of the following statements is correct?

- (A) $u(8) = 50$
- (B) $u(8) = 75$
- (C) $u(8) = 80$
- (D) $u(8) = 81$

9. A risk-averse farmer, whose preferences are characterized by the utility function $u(w) = \sqrt{w}$, is faced with the alternatives of growing grain or fruit on his fields.

Growing grain implies a total cost of €50 000. The revenue of grain sales is determined by the general situation on the national grain market — with a 70% chance, the farmer's revenue will be €90 000, and with 20% probability only €50 000. However, there is also a 10% chance that a revenue of €140 000 can be attained.

Growing fruit is more costly, as it totals to €100 000. Revenue here is dependent on the international supply of fruit — With a probability of 80% a minimum revenue of €110 000 can be realized, but there is also a 20% chance that the revenue of fruit sales amounts to €260 000.

How high is the farmer's expected profit in Euros, if he manages his fields according to the Bernoulli principle?

- (A) €17 000.
- (B) €37 000.
- (C) €40 000.
- (D) €90 000.

10. A team of scientists is due to spend six months in Antarctica carrying out research. One major piece of equipment they will be taking is subject to breakdowns caused by the sudden failure of a particular component. Because a failed component cannot be repaired, the team intends to carry a stock of spare units of the component, but it will cost them roughly \$3000 for each spare unit they take with them. However, if the equipment breaks down and a spare is not available, a new unit will have to be specially flown in, and the team will incur a total cost of \$5000 for each unit that is delivered in this way. An engineer who will be travelling with the team is certain that the number of spares required will not exceed three.

How many spares should the team carry with them if their objective is to minimize costs and they decide according to the Laplace rule?

- (A) 0
- (B) 1
- (C) 2
- (D) 3

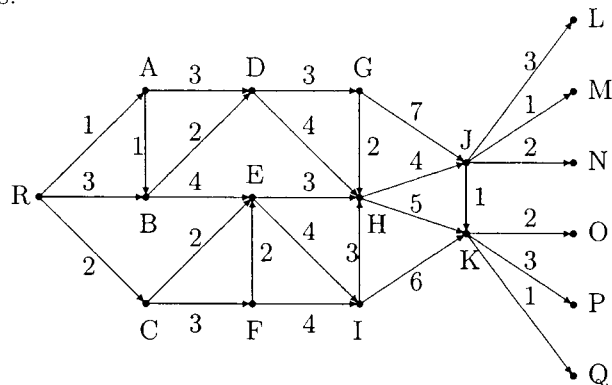
11. Consider the following decision matrix with profit payoffs:

| | θ_1 | θ_2 | θ_3 | θ_4 |
|-------|------------|------------|------------|------------|
| a_1 | 0 | 10 | 5 | 5 |
| a_2 | 9 | 0 | 1 | 0 |
| a_3 | 3 | 1 | 1 | 10 |
| a_4 | 5 | 2 | 0 | 5 |

A decision maker prefers a_4 to all other alternatives. This behavior is consistent with the

- (A) Maximin rule.
- (B) Hurwicz rule (with an optimism parameter of $1/2$).
- (C) Minimal-Regret rule.
- (D) Laplace rule.

12. Consider the sequential decision process given by the following graph. The decision process begins with the decision at R and ends at one of the outcomes L, M, N, O, P, or Q. The numbers along the edges represent the costs of taking the corresponding decisions.



The cost minimizing sequence of choices yields an aggregate cost of

- (A) 10
- (B) 11
- (C) 12
- (D) 13

13. A venture capitalist is considering the financing of two ventures. One is a proposal to market a generic brand of a drug whose patent is to expire shortly. This would yield a profit of \$100 000.

The other project would require an initial investment of €50 000 to develop a commercial application of a gene splicing technique. There is a 50% chance that the new technique will actually work successfully.

If the technique is successful, the resulting product could be marketed, requiring an additional investment of €100 000. With a 50% chance the new product will be successful, yielding an overall profit (i.e. taking into account the total investment) of \$350 000. If the product is not successful, it must be taken from the market, and there is no additional revenue.

If the development of the new gene-splicing technique turns out to be unsuccessful, then the generic brand could still be marketed afterwards, but there would then only be a 50% probability of success. If unsuccessful, the generic brand would have to be taken from the market.

Based on the given information, what is the optimal decision of the venture capitalist, assuming that he is risk neutral.

- (A) Immediately market the generic product.
- (B) Develop the gene splicing technique. If successful, market the resulting product, and, if unsuccessful, market the generic brand.
- (C) Develop the gene splicing technique. If successful, market the resulting product, and, if unsuccessful, exit both projects (i.e. without marketing the generic brand).
- (D) Toss a coin, due to indifference between marketing the generic product and developing the gene-splicing technique. technique.

14. In a multiple-choice test a candidate has to choose among four answers. Each candidate is either entirely ignorant and simply chooses an answer at random, or else is omniscient and knows the right answer for sure. If the proportion of omniscient candidates is $1/3$, what is the probability that a person who got the answer right was guessing?

(A) $1/5$

(B) $1/4$

(C) $1/3$

(D) $1/2$

15. Four well-shuffled cards, the Queen of clubs, the Queen of diamonds, the Queen of spades, and die Queen of hearts, are laid face down on the table. A person is offered the choice between the following alternatives:

Option 1: A randomly selected card will be turned over. If it is red, the person will win €200; if it is black the person will lose €200.

Option 2: A randomly selected card will be turned over. The person may then choose to quit the game or to continue. Quitting the game costs €30. If the person continues, one of the remaining three cards will be randomly selected and turned over. If it is red, the person will win €100; if it is black the person will lose €100.

Assuming that the person is risk neutral — which option is then the best choice?

- (A) Option 1 and 2 are equally attractive.
- (B) Option 2 with the choice of quitting if red appears.
- (C) Option 2 with the choice of quitting if black appears.
- (D) Option 1.

16. Consider the following two-player game in strategic form: The preferences of the players are given by the utility functions

$$u_1(x, y) = xy^2 - x^2 \quad \text{and} \quad u_2(x, y) = 8y - xy^2,$$

where $x \in \mathbb{R}$ denotes the strategy of player 1, and $y \in \mathbb{R}$ denotes the strategy of player 2. The Nash equilibrium of the game is then given by the strategy combination

- (A) $x = 1, y = 2$.
- (B) $x = 2, y = 1$.
- (C) $x = 2, y = 2$.
- (D) $x = 2, y = 3$.