Otto-von-Guericke-University Magdeburg
Faculty of Economics and Management
- Management Science -
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Management II (5072): Decision Theory

February 14, 2006

Last name: ..... **First name: .....** **Matriculation No.: .....**

Course: Decision Theory
Examiner: Prof. Dr. G. Wäscher

General remarks:

1. Write your name and matriculation number on this cover sheet and on every other sheet that has been issued to you.
2. Leave a minimum of 4 cm as correction space on the outside margin of each page.
3. Make sure that you have a complete copy of the test. The test consists of **6 assignments**, all of which have to be dealt with. It is not permitted to remove the retaining clip; doing so will be treated as fraudulent behaviour.
4. Please write legibly and number the pages which have been used. For each assignment, put down your answers on a separate sheet. Only pens with permanent ink may be used, while correction pens or ink erasers are not permitted. Make sure that you don't write in red.
5. Always make clear how you have determined your solution (solution path). Isolated solutions without traceable origin will not be accepted.
6. The following aids may be used: writing utensils, non-programmable pocket calculators without communicating and/or data processing functions, dictionaries (without any added remarks only).

Assignment 1 (15 points)

The following table comprises the data of a decision problem with four alternatives (a_1, a_2, a_3, a_4) and four states of nature (s_1, s_2, s_3, s_4). Note that the entries represent costs, which the decision maker wants to minimize.

	s_1	s_2	s_3	s_4
a_1	20	10	5	10
a_2	15	5	10	25
a_3	10	15	15	15
a_4	0	15	5	20

a) What would be the alternatives to be chosen if an optimistic, or alternatively, a pessimistic approach would be applied? What recommendation(s) would result from the application of the LAPLACE- and the MiniMax-Regret-Rule, respectively?

b) Give a formal definition of each of the above-mentioned rules!

Assignment 2 (15 points)

Describe the Direct-Rating Method for the determination of value functions!

Assignment 3 (23 points)

A decision maker has to choose between two investment projects. The net present value for each project is given in the following table:

investment	states of nature			
	s_1	s_2	s_3	s_4
project a	0	20	30	10
project b	10	0	10	30
probabilities	0.15	0.15	0.35	0.35

a) Give a formal, general definition of absolute dominance, state dominance and probability dominance!
b) For each of the projects, plot the corresponding Probability Function, Distribution Function and Risk-Profile.
c) Is one of the projects dominated by the other one? If so, which dominance principle is satisfied?

Assignment 4 (10 points)

Let the following utility function be given for a decision maker:

$$u(r) = -0.5 + 0.5 \cdot \sqrt{r} \quad , 1 \leqq r \leqq 9$$

(r: reward in TEURO). A lottery is offered to him, in which he receives 6 TEURO with a probability of 75%, and 3 TEURO with a probability of 25%, respectively.

What is the decision maker's Expected Reward, the Certainty Equivalent and the Risk Premium of this lottery?

Assignment 5 (10 points)

Consider the following linear multi-objective optimization problem:

$$
\begin{array}{rcrcrcrcrcrcrcrcl}
z_1 & = & 2x_1 & - & 1x_2 & + & 2x_3 & + & 2x_4 & & & & & & & \\
z_2 & = & 5x_1 & & & & & & & + & 1x_5 & - & 2x_6 & + & 3x_7 & \\
 & & 2x_1 & & & & & + & x_4 & + & x_5 & & & & & = & 10 \\
 & & & & 4x_2 & - & 2x_3 & + & 2x_4 & & & + & x_6 & & & = & 20 \\
 & & 2x_1 & + & x_2 & - & 2x_3 & & & & & & & + & x_7 & = & 10 \\
 & & x_1, & & x_2, & & x_3, & & x_4, & & x_5, & & x_6, & & x_7 & \geqq & 0
\end{array}
$$

$z_1 \rightarrow \text{Max!}$

$z_2 \rightarrow \text{Max!}$

Let

$$\mathbf{x}^* = \begin{pmatrix} 4 \\ 5 \\ 3 \\ 2 \\ 0 \\ 2 \\ 3 \end{pmatrix}$$

be a feasible solution of the system and

$$\mathbf{z}^* = \begin{pmatrix} 13 \\ 27 \end{pmatrix}$$

the corresponding vector of objective function values.

a) Formulate a model which can be used for testing whether $\mathbf{x}^*$ is dominated or not!
   Note: It is **not** requested that you determine a solution of this model.

b) What can be said in general about the optimal objective function values of the model and whether $\mathbf{x}^*$ is dominated or not?

Assignment 6 (27 points)

At a casino, a gambler plays against the bank. Four well-shuffled cards, the kings of clubs (black), diamonds (red), hearts (red) and spades (black), lay face down on the table. The following choice between three options is now offered to him:

Option 1: A randomly selected card will be turned over. If it is a red one, the gambler wins 100 EURO; if it is a black one, he loses 100 EURO.

Option 2: A randomly selected card will be turned over. At this point he may either continue or, alternatively, he can choose to pay 35 EURO which allows him to stop the game. If he continues, one of the remaining three cards will be randomly selected and turned over. If it is a red one, the gambler wins 100 EURO; if it is a black one, he loses 100 EURO.

Option 3: The gambler leaves the table without further gambling.

a) Plot a decision tree of the problem!
b) If the decision maker is risk neutral, what should he be doing?
c) Suppose instead that his utility function for money x is $u(x)=\ln(1+x/200)$ (for all relevant values). What should he do in this case if only Option 1 and Option 2 are available?