Examination: 5027
Economics III
Introduction to Econometrics
Winter Semester 2008 / 2009
Dr. John E. Brennan

You are allowed to use a non-programmable calculator (in accordance with the instructions given by the examination office) and a translating dictionary from your native language to English (without any notes written into it). **All** of the **twelve** (12) examination questions must be answered. This examination consists of **four** (4) pages and must be completed within 120 minutes.

NOTE: Answers that are NOT presented in a NEAT and ORDERLY manner, easy to read, will NOT be graded. All calculations should be rounded to four places following the decimal point (e.g., 15.6429)

Question 1 (15 points)

There are n different sets, $i = 1, 2, \ldots, n$, of the (k-1) explanatory variables, $j = 2, 3, \ldots, k$. The i^{th} set would be written as: $(X_2 = x_{i2}, X_3 = x_{i3}, \ldots, X_k = x_{ik})$. Consequently, the notation, $X_j = x_{ij}$, means that the (k-1) explanatory variables are observed with their i^{th} value. Consider the (n x n) covariance matrix of the n univariate conditional random variables, $(Y|X_j = x_{ij})$:

$$
\underset{(n \times n)}{\mathrm{Cov}\,(Y \mid \mathbf{X})} =
\begin{array}{c}
\\ 1 \\ 2 \\ \\ i \\ \\ n
\end{array}
\begin{bmatrix}
\scriptstyle 1 & \scriptstyle 2 & \cdots \; \scriptstyle i & \cdots \; \scriptstyle n \\
\mathbf{V(Y|X_j = x_{1j})} & C(Y|X_j = x_{1j}, Y|X_j = x_{2j}) & C(Y|X_j = x_{1j}, Y|X_j = x_{ij}) & C(Y|X_j = x_{1j}, Y|X_j = x_{nj}) \\
C(Y|X_j = x_{2j}, Y|X_j = x_{1j}) & \mathbf{V(Y|X_j = x_{2j})} & C(Y|X_j = x_{2j}, Y|X_j = x_{ij}) & C(Y|X_j = x_{2j}, Y|X_j = x_{nj}) \\
 & & \ddots & \\
C(Y|X_j = x_{ij}, Y|X_j = x_{1j}) & C(Y|X_j = x_{ij}, Y|X_j = x_{2j}) & \mathbf{V(Y|X_j = x_{ij})} & C(Y|X_j = x_{ij}, Y|X_j = x_{nj}) \\
 & & & \ddots \\
C(Y|X_j = x_{nj}, Y|X_j = x_{1j}) & C(Y|X_j = x_{nj}, Y|X_j = x_{2j} & C(Y|X_j = x_{nj}, Y|X_j = x_{ij}) & \mathbf{V(Y|X_j = x_{nj})}
\end{bmatrix}
$$

a. Completely explain what we mean by the term "Hetroscedasticity" and what it tells us in a model where Y = Savings Rate and X = Family Income.

b. Explain the conditions that are necessary for the covariances in this matrix to be exchanged for correlation coefficients.

c. Completely define the First-Order Autoregressive Process, AR(1). With AR(1), What is the value of $\mathrm{Corr}\,[(Y \mid X_j = x_{3j}), (Y \mid X_j = x_{8j})]$?

Question 2 (10 points)

Consider the discrete random variables (X, Y) with joint pmf: f (x, y) where X= x and Y= y.

a. Explain in detail how the probabilities associated with the conditional random variables $(Y \mid X = x)$ relate to the joint probabilities, f (x, y).

b. When are the probabilities associated with the conditional random variables $(Y \mid X = x)$ equal to the marginal probabilities of Y, $f_2(y)$.

Please turn to page 2

Question 3 (10 points)

Consider the continuous univariate random variable X= x ~ Rectangular with $a < b$,

$$f(x) = 1 / (b - a) \text{ and } F(x) = (x - a) / (b - a) \text{ for } a \le x \le b.$$

Calculate the following using a = 4 and b = 8.

a. $\Pr\{3.0 \le X = x \le 5.0\}$

b. $\Pr\{3.0 \le X = x \le 5.0 \mid X = x \le 6\}$

Question 4 (10 points)

Given the discrete bivariate joint probability distribution, $(X, Y) \sim f(x, y)$.

Y \ X	1.5	3.5	5.5	$f_2(y)$
0.4	0.045	0.080	0.045	
0.6	0.120	0.120	0.120	
0.8	0.135	0.200	0.135	
$f_1(x)$				1.00

a. Calculate C(X, Y). Is Y stochastically independent of X? Explain your answer in complete detail.

b. Calculate E(Y) and $E(Y \mid X = 3.5)$. Explain your answer in complete detail.

Question 5 (10 points)

You are writing a Master's thesis in the Department of Economics at the Martin-Luther-Universität Halle-Wittenberg. The relationship between private consumption and family income in eastern Germany is being studied. Annual data has been obtained from 1978 to 2007 (30 observations on consumption and income). Your study should take into account the change in structure that occurred as a result of German political reunification in 1989.

a. Explain in detail a BLP estimation procedure that would allow the estimated "level" of consumption to be different before and after the structural change.

b. Explain in detail a BLP estimation procedure that would allow the estimated marginal propensity to consume (mpc) to be different before and after the structural change.

Question 6 (10 points)

The Runs Test (Geary Test) is a nonparametric test that can be used to test for the presence of autocorrelation in a sample.

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a. Explain the rationale behind the Runs (Geary) Test.

$$E(\delta) = [(2\, n_1 n_2) / (n_1 + n_2)] + 1$$

$$V(\delta) = \frac{2\, n_1 n_2 (2\, n_1 n_2 - n_1 - n_2)}{(n_1 + n_2)^2 (n_1 + n_2 - 1)}$$

b. Can we accept the null hypothesis in this case? Explain your answer fully.

Please turn to page 3

Question 7 (10 points)

Consider a random sample of 15 annual observations (y_t, $x_{t\,2}$, $x_{t\,3}$). The following matrices were calculated:

det **X' X** = 306223760 Calculate estimates of the BLP = E*(y | **X**) = β **X**

$$\mathbf{X'\,X} = \begin{bmatrix} 15 & 31895 & 120 \\ 31895 & 68922513 & 272144 \\ 120 & 272144 & 1240 \end{bmatrix} \quad (\mathbf{X'\,X})^{-1} = \begin{bmatrix} 37.232772 & -0.0225081 & 1.3367066 \\ -0.0225081 & 0.0000137 & -0.0008319 \\ 1.3367066 & -0.0008319 & 0.054035 \end{bmatrix}$$

$$\mathbf{X'\,y} = \begin{bmatrix} 29135 \\ 62905821 \\ 247934 \end{bmatrix} \qquad \Sigma_t\,(y_t - c_1 - c_2\,x_{t\,1} - c_3\,x_{t\,2})^2 = 1976.85539$$

(based on the correct values of the parameter estimates)

a. Calculate the vector **c**.

b. Calculate the Var-Cov matrix for the vector **c**.

Question 8 (10 points)

Given an annual time-series sample of the 34 observations on the random variables (Y, X, W) for t = 1972, 1973, …, 2005, an estimate of the correlation coefficient, $\bar{\rho}$, between the univariate conditional random variables ($Y \mid X_t = x_t$, $W_t = w_t$) and ($Y \mid X_{t-1} = x_{t-1}$, $W_{t-1} = w_{t-1}$) can be obtained using the Durbin-Watson d statistic:

$$d = 2\,(1 - \bar{\rho})\,.$$

a. If you obtained the Durbin-Watson statistic, d = 2.7135, with n = 34, k' = 2, $d_L = 1.333$ and $d_U = 1.580$. Perform the Durbin-Watson test.

b. Explain WHY the Durbin-Watson d statistic is a number near 2.0 when null hypothesis is not rejected. Why does $0 \le d \le 4.0$?

Question 9 (15 points)

Given a sample of 32 yearly observations (y_t, x_{t2}), the following matrices were calculated:

$$(\mathbf{X'\,X})^{-1} = \begin{bmatrix} 1.606345267 & -0.01688205 \\ -0.01688205 & 0.00018094 \end{bmatrix} \qquad \mathbf{X'\,y} = \begin{bmatrix} 2995.62 \\ 283964.668 \end{bmatrix}$$

Note: the following values were calculated using the correct c_1 and c_2 estimates.

$$\Sigma_t\,(y_t - c_1 - c_2\,x_{t2})^2 = 204.804524 \text{ and } s^2(Y) = 65.1762$$

$$\Sigma_s\,[(y_s - c_1 - c_2\,x_{s2})(y_{s-1} - c_1 - c_2\,x_{s-12})] = 182.265061$$

$$\Sigma_s[(y_s - c_1 - c_2\,x_{s2}) - (y_{s-1} - c_1 - c_2\,x_{s-12})]^2 = 28.299548$$

$$\Sigma_s[(y_s - c_1 - c_2\,x_{s2}) - (y_{s-1} - c_1 - c_2\,x_{s-12})] = -\,0.900113$$

$\Sigma_t(c_1 + c_2\,x_t - \bar{y})^2 = 1522.516$, where t = 1, 2, …, T and s = 2, 3, …, T.

a. Calculate the values in the (2 x 1) vector **c**.

b. Calculate the Durbin-Watson d statistic and test the H_o: There is no positive or negative autocorrelation present.

[n = 32, k' = 1: $d_U = 1.489$ and $d_L = 1.352$]

c. Calculate R^2

Question 10 (10 points)

Sample estimates, **c**, of the (k x 1) vector of population parameters can be made subject to q linear constraints imposed by:

$$\mathbf{R}\,\mathbf{c} = \mathbf{r}$$

Where **R** is a matrix (q x k), $q \leq k$ and **r** is a (q x 1) vector

if:

$$BLP = c_1 + c_2\,x_2 + c_3\,x_3 + c_4\,x_4 + c_5\,x_5$$

Write down the matrix **R** and the vector **r** that would impose the following constraints:

a. $c_2 = c_3$ and $c_4 = c_5$

b. $c_2 - 7\,c_3 = 6\,c_4$

Question 11 (10 points)

Stochastic independence and Mean independence both imply that $C(X, Y) = 0$.

a. Explain the difference between stochastic and mean independence.

b. If X and Y are stochastically independent, explain the implication for the CEF and the BLP.

Question 12 (10 points)

At the faculty of Economics and Management, 75% of the baccalaureate students in the English language program take the course Economics I, 65% of them take the course Economics II, and 85% of them take at least one of the two courses.

a. What proportion of students takes both courses?

b. What is the probability that a student will take Economics II given that he or she has taken Economics I?

c. Are the decisions made by students to take these two courses independent?

This is the end of the examination.

<u>GOOD LUCK !!</u>

(Viel Glück! Alles Gute!)