Original

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Examination in Mathematics I

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Working time: 120 minutes

The derivation of the results must be given clearly. The statement of the result only is not sufficient.

Tools:

- pocket calculator
- printed collection of formulas
- script "Mathematics for Students of Economics and Management"

It is not allowed to use mobile phones.

Problems:

1. Given is the complex number $z_1 = -1 + \sqrt{3}i$. Determine all solutions of the equation $z^4 = z_1$ in the form $z = a + bi$.

2. A construction firm wants to buy a building site and has the choice between three different payment schedules:

   (a) Pay DM 67,000 in cash.

   (b) Pay DM 12,000 per year for 8 years, where the first installment is to be paid at once.

   (c) Pay DM 22,000 in cash and thereafter DM 7,000 per year for 12 years, where the first installment is paid after one year.

   Determine which schedule is least expensive if the interest rate is 11.5% and the firm has at least DM 67,000 available to spend in cash.

3. Given is the function $f(x) = \sqrt{x+1} + \sqrt{x-1}$.

   (a) Find domain and range.

   (b) Determine intervals of monotonicity and concavity respectively convexity.

   (c) Give reasons for the existence of the inverse function f^{-1}. Find a formula for f^{-1} and its domain and range.

4. A firm produces a commodity using only one input. Let

$$f(x) = (x-1)^{\frac{1}{3}} + 1$$

for $x \geq 0$ be the production function when x units of the input are used.

   (a) Suppose that the price per unit of the commodity is 1 and that the price the firm must pay per unit of the input is p. The profit is then

$$\pi(x) = 1 \cdot f(x) - px.$$

   Suppose that $x_m \geq 0$ maximizes $\pi(x)$ for the given value of $p \geq 0$. Find x_m expressed in terms of p and prove that it is a local maximum point.

   (b) Let $p = 3$. Determine the global maximum of $\pi(x)$ for $x \geq 0$.

   (c) Find the zeros of the equation $\pi(x) = 0$. For which values of p are there three real zeros which need not to be different?

5. Given is the function

$$f(x) = e^{\frac{x^3}{3} - 2x}.$$

Decide whether $f(x)$ is elastic for $x > \sqrt{2}$.

6. Find respectively evaluate the following integrals

   (a) $\int x^2 e^{x+2}\, dx$

   (b) $\int \frac{\sqrt{x}}{1+\sqrt{x}}\, dx$

   (c) $\int_1^\infty \left(\frac{k}{x} - \frac{k^2}{1+kx}\right) dx \qquad (k > 0\,;\text{constant}).$