

Fakultät für Mathematik
Institut für Mathematische Optimierung
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Examination in Mathematics I
(20.07.2001)

Working time: 120 minutes

The derivation of the results must be given clearly. The statement of the result only is not sufficient.

Tools:

- pocket calculator
- printed collection of formulas
- printed script “Mathematics for Students of Economics and Management”

It is not allowed to use mobile phones.

Distribution of points obtainable for the problems:

problem	1	2	3	4	5	6	sum
points	7	8	8	7	10	10	50

Problems:

1. Given are the complex numbers

$$z_1 = 4i \quad \text{and} \quad z_2 = 2 - 2\sqrt{3}i.$$

- (a) Find the cartesian form $a + bi$ of the number $\frac{z_1}{z_2}$.
- (b) Determine the power $z_3 = (z_2)^8$ as $z_3 = a + bi$.
2. (a) Let $\{b_n\}, n \in \mathbb{N}$, be a geometric sequence with the terms $b_1 = -10$ and $b_4 = \frac{16}{25}$. Which of the terms is the first with an absolute value less than $\frac{1}{100}$?
- (b) Two economists, Mr. A and Mrs. B started their jobs on January 1, 1990. During the first year Mr. A got a fixed salary of 4.000 DM every month and his salary has been raised by 5% every year up to now. Mrs. B earned in 1990 53.000 DM and her salary has been raised by 3% every year. What was the salary of Mr. A and Mrs. B in 2000? Who of them did receive more money over the years up to the end of 2000?
3. Let $P(x) = x^4 - 2x^3 + 17x^2 - 32x + 16$.

- (a) Determine all real and complex zeros of $P(x)$.
- (b) The function $f(x)$ is defined as

$$f(x) = \frac{(x-1)^2(x+16)}{P(x)}.$$

Give reasons for the existence of a discontinuity at $x_0 = 1$ and find the limit $\lim_{x \rightarrow 1} f(x)$.

4. Consider the function

$$f(x) = xe^{\frac{1}{8}x^2 - x}, \quad x > 0.$$

Check for what x the rate of change $\rho_f(x)$ is greater than one.

5. A function is given by the formula

$$f(x) = \frac{2x^2 + 3x - 2}{x - 2}.$$

- (a) Find the domain of f , extreme points and inflection points.
- (b) Investigate monotonicity and concavity / convexity.

6. Let

$$f(x) = (2x - 6) \ln(x - 3).$$

- (a) Find domain, zeros and $\lim_{x \rightarrow 3} f(x)$.

- (b) Evaluate $\int_4^6 f(x) dx$.

Examination in Mathematics I - Solutions

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1. (a) $z = \frac{1}{2}(-\sqrt{3} + i)$
(b) $z_3 = 4^7(-2 - 2\sqrt{3}i) = 32,765(-1 - \sqrt{3}i)$
2. (a) $|b_n| < \frac{1}{100}$ for $n \geq 9$. That means the answer is b_9 .
(b) Salary of Mr. A in 2000: 78,186.94 DM,
salary of Mrs. B in 2000: 71,227.57 DM
Mr. A received 681,925.78 DM which was more than 678,813.17 for Mrs. B
3. (a) zeros: $x_1 = 1, \quad x_2 = 1, \quad x_3 = 4i, \quad x_4 = -4i$
(b) The function value $f(1)$ does not exist, since $x = 1$ is a zero of the polynomial in the denominator. Therefore $f(x)$ is not continuous at $x_0 = 1$.
(c) $\lim_{x \rightarrow 1} f(x) = 1$
4. $0 < x < 4 - 2\sqrt{3} \quad \vee \quad 4 + 2\sqrt{3} < x$
5. (a) Domain $D_f = \{x \in \mathbb{R} \mid x \neq 2\}$
extreme points:
local maximum at $x_1 = 2 - \sqrt{6}$, local minimum at $x_2 = 2 + \sqrt{6}$
no inflection point
 $f'(x) = \frac{2x^2 - 8x - 4}{(x - 2)^2}, \quad f''(x) = \frac{24}{(x - 2)^3}$

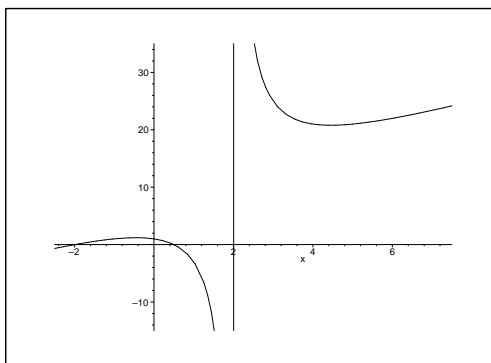


Abbildung 1: graph of the function

- (b) $f(x)$ is increasing for $x < x_1 \vee x > x_2$ and decreasing for $x_1 < x < 2 \vee 2 < x < x_2$
 $f(x)$ is concave for $x < 2$ and convex for $x > 2$.

6. (a) $D_f = \{x \in \mathbb{R} \mid x > 3\}$, zeros: $x_1 = 4$, $\lim_{x \rightarrow 3} f(x) = 0$

(b) $\int_4^6 (2x - 6) \ln(x - 3) dx = 9 \ln 3 - 4$; (partial integration)

Additionally there is the graph of the function $f(x) = 2(x - 3) \ln(x - 3)$ with the area which is to compute with the integral $\int_4^6 (2x - 6) \ln(x - 3) dx$

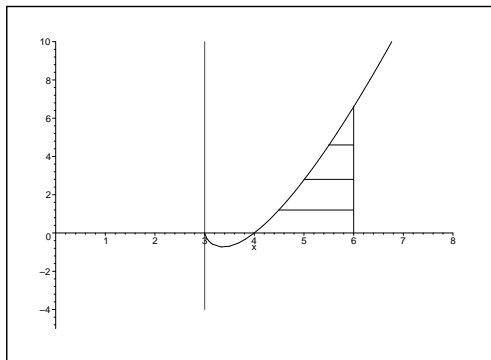


Abbildung 2: graph