

Fakultät für Mathematik
Institut für Mathematische Optimierung
Prof. Dr. F. Werner

Examination in Mathematics I
(23.07.2003)

Working time: 120 minutes

The derivation of the results must be given clearly. The statement of the result only is not sufficient.

Tools:

- pocket calculator
- printed collection of formulas
- printed script "Mathematics for Students of Economics and Management"

It is not allowed to use mobile phones.

Distribution of points obtainable for the problems:

problem	1	2	3	4	5	6	sum
points	7	7	9	10	7	10	50

Problems:

1. Given are the complex numbers

$$z_1 = 2 - 2i \quad \text{and} \quad z_2 = 3 + 2i.$$

(a) Find $z_3 = z_1 - z_2$ and $z_4 = \frac{z_1}{z_2}$. Give the results in cartesian form.

(b) Find all third roots of z_1 .

2. Your grandparents have saved money for you. They offer the following choices:

(a) You get 20 000 € now.

(b) You get 2 500 € every year for ten years beginning now.

(c) Now you get 10 000 € and then you will get 1 300 € at the end of each of the following ten years.

What is best for you when the interest rate is 5% ?

3. Consider the function $f : D_f \rightarrow \mathbb{R}, D_f \subseteq \mathbb{R}$, with

$$f(x) = \frac{x^2 - 1}{x^2(x^2 + 2)}.$$

(a) Find the domain of f and the zeros.

(b) Find $\lim_{x \rightarrow \infty} f(x)$ and $\lim_{x \rightarrow 0} f(x)$.

(c) Is f an even or odd function?

(d) Does $f^{-1}(x)$ exist? Give reason[s] for your answer.

4. Given is the function $f : D_f \rightarrow \mathbb{R}, D_f \subseteq \mathbb{R}$, with

$$f(x) = (x + 2)(x - 2)^3$$

(a) Find extreme points and check monotonicity.

(b) Find inflection points and check convexity/concavity.

5. Consider the function $f : D_f \rightarrow \mathbb{R}, D_f \subseteq \mathbb{R}$, with

$$f(x) = e^{(x+1)^2} \quad \text{and} \quad x > 0.$$

(a) Find the elasticity $\varepsilon_f(x)$ of this function.

(b) Show that $f(x)$ is elastic at $x_0 = 1$.

(c) Is $f(x)$ elastic for all $x > 0$ or does elasticity change? Check it.

6. (a) Find

$$\int \frac{\sin x}{\cos^2 x} dx$$

(b) Evaluate

$$\int_0^{\infty} x e^{-x} dx$$