

Original

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Examination in Mathematics A
(24.7.98)

Working time: 120 minutes

The derivation of the results must be given clearly. The statement of the result only is not sufficient.

Tools:

- pocket calculator
- printed collection of formulas
- lecture notes

Problems:

1. The polynomial $P_3(x) = x^3 - 8x + 32$ has a (real) zero at $x_1 = -4$.
 - (a) Determine all remaining zeros of the polynomial.
 - (b) Find the trigonometric (polar) forms of these zeros
 - (c) Determine the square roots of these zeros and find their cartesian forms $a + bi$.
2. Given are the three vectors

$$a^{(1)} = \begin{pmatrix} 2 \\ 3 \\ 1 \end{pmatrix}, a^{(2)} = \begin{pmatrix} 1 \\ -1 \\ 3 \end{pmatrix} \text{ and } a^{(3)} = \begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix}$$

- (a) Is it possible to express the vector $b = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$ as a linear combination of the vectors $a^{(1)}, a^{(2)}, a^{(3)}$?

- (b) Do the vectors $a^{(1)}$, $a^{(2)}$, $a^{(3)}$ constitute a basis in \mathbb{R}^3 ?
Give the reasons for your opinion.

3 Given is the matrix

$$A = \begin{pmatrix} 1 & 0 & -1 & 2 \\ 0 & t & 2 & -5 \\ 1 & 1 & 0 & -1 \\ 2 & -1 & -2 & -3 \end{pmatrix}$$

- (a) Find conditions for parameter t such that the inverse matrix does not exist.
(b) Find all values of t for which matrix A is positive definite
4. Given is the following matrix equation

$$A \cdot X = X - B$$

- (a) What are the dimensions of matrices A and X when matrix B has the dimension (p, q) ?
(b) Find the solution X of the matrix equation.
(c) Calculate the solution X , where $A = \begin{pmatrix} 3 & \mu \\ 10 & 6 \end{pmatrix}$ and $B = \begin{pmatrix} 3 & 1 \\ -1 & 2 \end{pmatrix}$.
(d) For which values of parameter μ does the matrix equation not have a unique solution?

5. Given is the following linear system of equations:

$$\begin{array}{rcccc} 2x_1 & - & x_2 & - & x_3 & - & 2x_4 & = & 1 \\ 2x_1 & & & - & 2x_3 & + & 7x_4 & = & -3 \\ -3x_1 & + & 2x_2 & + & 2x_3 & + & x_4 & = & 3 \\ 5x_1 & - & x_2 & - & 3x_3 & + & 2x_4 & = & 3 \end{array}$$

- (a) Calculate the general solution of the system.
(b) Find an interval for the variable, which can be chosen freely, such that all remaining variables have only nonnegative values.

- (c) Specify two different basic solutions.

6 A self-employed interpreter and translator has a monthly working time of at most 200 hours. He wants to divide it in such a way that the time for interpreting is at least the same as the time for translating because he gets 30 DM per hour for interpreting and 20 DM per hour for translating. But unfortunately the orders for translating per month amount to at most 120 hours and he has to translate at least 50 hours because of contracts he made.
Formulate a linear programming problem and determine an optimal solution graphically