Original

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Examination in Mathematics I
(23.7.99)

Working time: 120 minutes

The derivation of the results must be given clearly. The statement of the result only is not sufficient.

Tools:

- pocket calculator
- printed collection of formulas
- either script "Mathematics for Students of Economics and Management" or own lecture notes (without solved exercises)

Problems:

1. Given are the complex numbers
   $z_1 = 2 - 2i$ und $z_2 = (i-2)^2 + 3i$.

   (a) Find the cartesian form $a + bi$ of the complex numbers $\frac{z_1}{z_2}$ and $\frac{z_2}{z_1}$.

   (b) Find the moduli r of the numbers $\frac{z_1}{z_2}$ and $\frac{z_2}{z_1}$.

   (c) Determine all solutions $z = a + bi$ of the equation $z^3 = 2 - 2i$.

2. Given are the vectors

$$a^{(1)} = \begin{pmatrix} 2 \\ 0 \\ -1 \end{pmatrix}, \quad a^{(2)} = \begin{pmatrix} 1 \\ -3 \\ 1 \end{pmatrix}, \quad a^{(3)} = \begin{pmatrix} -1 \\ -1 \\ 1 \end{pmatrix}.$$

   (a) Do $a^{(1)}, a^{(2)}, a^{(3)}$ constitute a basis in $\mathbb{R}^3$?

(b) For which values of α is it possible to express vector

$b = \begin{pmatrix} \alpha \\ 1 \\ 2 \end{pmatrix}$ as a linear combination of $a^{(1)}, a^{(2)}, a^{(3)}$?

(c) Express vector b for the α-value found in (b) as a linear combination of $a^{(1)}, a^{(2)}, a^{(3)}$.

3. The linear mapping

$$\begin{pmatrix} y_1 \\ y_2 \\ y_3 \end{pmatrix} = \begin{pmatrix} \lambda & 1 & 2 \\ -2 & 0 & 2 \\ 1 & 1 & 1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$$

assigns to each

$$x = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} \in \mathbb{R}^3 \text{ a unique } y = \begin{pmatrix} y_1 \\ y_2 \\ y_3 \end{pmatrix} \in \mathbb{R}^3.$$

(a) Find the inverse mapping for $\lambda = -1$.

(b) For which λ the inverse mapping does not exist?

4. Find conditions for x such that matrix

$$\begin{pmatrix} x & 1 & -1 & 2 \\ 1 & -2 & 1 & 0 \\ -1 & 1 & -1 & 0 \\ 2 & 0 & 0 & -2 \end{pmatrix}$$

is negative definite.

5. Check the consistence of the following system of linear equations as a function of μ:

$$\begin{aligned} -x_1 + x_2 + x_3 - x_4 &= 4 \\ 2x_1 - x_2 - x_3 - 2x_4 &= 1 \\ -x_1 + 2x_2 + 2x_3 - 5x_4 &= 13 \\ x_2 + x_3 - 4x_4 &= \mu \end{aligned}$$

Find the general solution for those values of μ for which the system is consistent.

6. Solve the following system of linear inequalities graphically:

$$\begin{aligned} 3x_1 + 4x_2 &\leq 24 \\ 4x_1 + 3x_2 &\geq 12 \\ x_1 - x_2 &\leq 2 \end{aligned}$$

$$x_1, x_2 \geq 0.$$

Evaluate the extreme points exactly and find the general solution.