Original

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Examination in Mathematics A

(13.2.98)

Working time: 120 minutes

The derivation of the results must be given clearly. The statement of the result only is not sufficient.

Tools:

- pocket calculator
- printed collection of formulas
- lecture notes

Problems:

1.(a) Given is the complex number $w = 3 + 2i$. Determine the complex number $z = a + bi$ such that $w \cdot z = 4$.

(b) Find all complex numbers z for which the equation $z^3 = -27i$ holds.

2.(a) Do the 3 vectors

$$a^{(1)} = \begin{pmatrix} 2 \\ 0 \\ 1 \end{pmatrix}, \quad a^{(2)} = \begin{pmatrix} -2 \\ 1 \\ -1 \end{pmatrix} \quad \text{and} \quad a^{(3)} = \begin{pmatrix} -4 \\ 1 \\ 3 \end{pmatrix}$$

constitute a basis in $\mathbb{R}^3$?

(b) Express vector $b = \begin{pmatrix} 0 \\ 1 \\ 5 \end{pmatrix}$ as a linear combination of the vectors $a^{(1)}, a^{(2)}, a^{(3)}$.

(c) Find all basises for the 3-space which include b and vectors from above.

3. Find conditions for a such that

$$A = \begin{pmatrix} 2 & 1 & 0 & 0 \\ 1 & a & 1 & 0 \\ 0 & 1 & 3 & 1 \\ 0 & 0 & 1 & 2 \end{pmatrix}$$

is positive definite.

4. Calculate the solution X of the following matrix equation:

$$A \cdot X \cdot B = I$$

where

$$A = \begin{pmatrix} -5/3 & 13/6 & -1/2 \\ 4/3 & -5/6 & 1/2 \\ 2/3 & -1/6 & 1/2 \end{pmatrix}, B = \begin{pmatrix} 3 & 4 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 2 \end{pmatrix} \text{ and } I = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

5. A firm produces 3 goods G_1, G_2, G_3 which are sold at 3 different places P_1, P_2, P_3. Let u_{ij} be the number of units of G_j which can be sold at place P_i, $i, j = 1, 2, 3$.
These u_{ij} values are given in the following table:

	G_1	G_2	G_3
P_1	2	1	3
P_2	2	2	4
P_3	4	0	4

We denote the prices per unit of G_1, G_2, G_3 by x_1, x_2, x_3. What prices should be chosen, when the sales amount to 24 for P_1, 32 for P_2 and 32 for P_3 ?

(a) Translate the problem into a system of linear equations.
(b) Find the general solution of the system.
(c) How does the solution change, if the prices x_2 and x_3 should be equal?

6. In a department store the departments for sports (S) and for musics (M) must be arranged on the same storey. The whole area is not larger than 300 m^2. The area of the sports department should be at most 220 m^2, but more than two times the amount of the M - department plus 30 m^2 is impossible.
What area combinations are possible?

(a) Translate the problem into a system of linear inequalities.
(b) Find all basic feasible solutions and calculate the general solution.
(c) Determine the set of feasible solutions graphically.