

Original

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Examination in Mathematics A
(17.2.99)

Working time: 120 minutes

The derivation of the results must be given clearly. The statement of the result only is not sufficient.

Tools:

- pocket calculator
- printed collection of formulas
- lecture notes

Problems:

1. Determine all solutions of the equation

$$z^3 = -8 + 8i.$$

(Give the solutions in the cartesian form $z = a + bi$.)

2. Given is the function $f(x) = (x^5 - x^4 + x^3 - x^2) \ln x^2$.

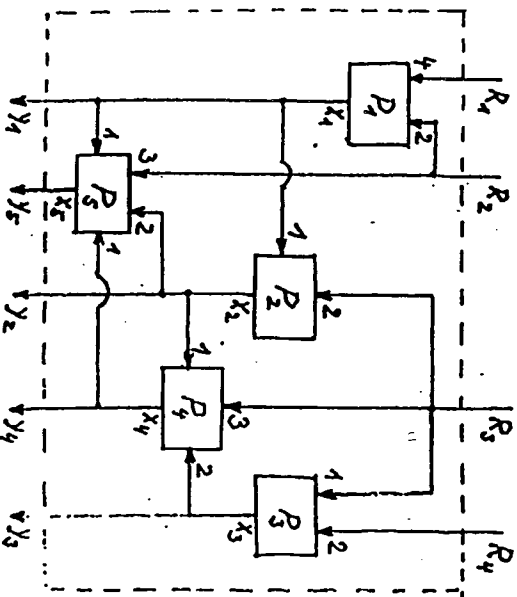
- (a) Find the domain.
- (b) Find all zeros of the function.
- (c) Is f an even or odd function? Justify your statement.
- (d) Does the inverse function f^{-1} exist? Give reason(s) for your answer.
- (e) Find $f \circ g$ when $g(x) = e^x$ and simplify as far as possible.

3. Given is a set $M = \{a^{(1)}, a^{(2)}, a^{(3)}, a^{(4)}\}$ containing as elements the vectors

$$a^{(1)} = \begin{pmatrix} -1 \\ 1 \\ -2 \end{pmatrix}, \quad a^{(2)} = \begin{pmatrix} 5 \\ 1 \\ 0 \end{pmatrix}, \quad a^{(3)} = \begin{pmatrix} 3 \\ 1 \\ -4 \end{pmatrix}, \quad a^{(4)} = \begin{pmatrix} 1 \\ 0 \\ 2 \end{pmatrix}$$

Find all bases for the 3-space \mathbb{R}^3 constituted by vectors of set M .

4. A firm produces by means of 4 raw materials R_1, R_2, R_3 and R_4 5 products P_1, \dots, P_5 , where some of these products are also taken as intermediate products. The relations are given in the following graph:



The numbers besides the arrows describe how many units of R_j , resp. P_i are necessary for one unit of P_j , $j = 1, \dots, 5$. $x^T = (x_1, x_2, x_3, x_4, x_5)$ describes the produced units (total output) of P_i and $y^T = (y_1, y_2, y_3, y_4, y_5)$ denotes the final demand (export) for the output of P_i .

- (a) Find the relation between x and y .

2

- (b) Calculate the final demand when the total output is given by

$$x^T = (200, 120, 100, 30, 10).$$

- (c) Let $y^T = (50, 40, 30, 20, 10)$ be the given final demand vector. Find for this case the production vector x and calculate the required units of raw materials.

5. Find conditions for α such that matrix

$$A = \begin{pmatrix} -1 & \alpha & 0 & 1 \\ \alpha & -2 & 0 & 0 \\ 0 & 0 & -3 & 0 \\ 1 & 0 & 0 & -4 \end{pmatrix}$$

is negative definite?

6. Given is the following linear system of equations

$$\begin{aligned} 2x + y + z &= 0 \\ -2\lambda x + \lambda y + 9z &= -18 \\ 2x + 2y + z &= 1 \end{aligned}$$

- (a) Calculate the solution for $\lambda = 1$.
 (b) Check the consistence of the system as a function of the parameter λ .

3