

Fakultät für Mathematik
Institut für Mathematische Optimierung
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Examination in Mathematics I
(11.02.2000)

Working time: 120 minutes

The derivation of the results must be given clearly. The statement of the result only is not sufficient.

Tools:

- pocket calculator
- printed collection of formulas
- script "Mathematics for Students of Economics and Management"

It is not allowed to use mobile phones.

Problems:

1. Solve for x when

$$(a) \frac{\frac{a}{a-x} - \frac{x}{a+x}}{\frac{a}{a+x} + \frac{x}{a-x}} = x$$

$$(b) 3 \lg(9x^2) - 2 \lg(3x) = 4$$

2. Given are the complex numbers

$$z_1 = -2 + 2i \quad \text{and} \quad z_2 = 3 + i$$

(a) Find the cartesian form $a + bi$ of the number $\frac{z_1}{z_2}$.

(b) Determine all solutions $z = a + bi$ of the equation $z^3 = z_1$.

3. A transport company has taken up a loan of 150,000 DM for buying a special-purpose vehicle. The interest rate is 6.5% annually.
- How many years does the firm repay when the loan is paid off in equal installments of 15,000 DM every year (ordinary annuity)?
 - Calculate the yearly payments required to pay off the loan after 5 years.

4. Given is the function f with the function values

$$f(x) = \frac{4x + 5}{(x - 1)^4}$$

- Find the domain of f .
- Determine zeros and extreme points.
- For which x is the function concave?

5. A monopolistic firm will be assumed to have a total-cost function

$$K(x) = k \cdot x + 45$$

where x is the output (the number of units produced) and k are the production costs per unit. The demand x which is equal to the output depends on price p only:

$$x = f(p) = 400 - 20p.$$

- Formulate the profit function $G = x \cdot p - K(x)$ in terms of the price variable p .
- Assume $k = 4$ and find the profit-maximizing price level p^* .
- Find the optimal price p^* and the maximal profit as a function of k .
- Check for which k the maximal profit is at least 1,400.

6. Find respectively evaluate the following integrals

$$(a) \int x^3 \ln x \, dx$$

$$(b) \int_1^{\infty} \frac{\ln x}{x^2} \, dx$$