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Examination in Mathematics II
(18.07.2000)

Working time: 120 minutes

The derivation of the results must be given clearly. The statement of the result only is not sufficient.

Tools:

- pocket calculator
- printed collection of formulas
- script "Mathematics for Students of Economics and Management"

It is not allowed to use mobile phones.

Problems:

1. Given are the four vectors

$$a^{(1)} = \begin{pmatrix} 1 \\ -3 \\ 2 \end{pmatrix}, \quad a^{(2)} = \begin{pmatrix} -2 \\ 7 \\ -1 \end{pmatrix} \quad a^{(3)} = \begin{pmatrix} 5 \\ -12 \\ 14 \end{pmatrix}, \text{ and } a^{(4)} = \begin{pmatrix} 3 \\ -10 \\ 3 \end{pmatrix}.$$

(a) Do the vectors $a^{(1)}$, $a^{(2)}$, $a^{(3)}$ constitute a basis in $\mathbb{R}^3$? Give the reasons for your opinion and express the vector $a^{(4)}$ as a linear combination of the vectors $a^{(1)}$, $a^{(2)}$, $a^{(3)}$ if possible.

(b) Find other basises for the $\mathbb{R}^3$ including $a^{(4)}$ and vectors from above.

2. Given is an open Input-Output-Model (Leontief Model) with

$$A = \begin{pmatrix} \frac{1}{5} & \frac{1}{5} & 0 \\ \frac{2}{5} & \frac{3}{5} & \frac{1}{5} \\ \frac{3}{5} & \frac{2}{5} & \frac{1}{5} \end{pmatrix}$$

Let x be the total vector of goods produced and y the final demand vector with $y^T = (2 \quad 0 \quad 1)$.

(a) Find the linear mapping $x \mapsto y$.

(b) Calculate x by means of matrix inversion.

3. Consider the system

$$\begin{array}{rcrcrcrcrcr} x_1 & & & + & 2x_3 & & & + & 3x_5 & = & 3 \\ & & x_2 & - & 2x_3 & + & x_4 & - & x_5 & = & q \\ -x_1 & & & - & 3x_3 & + & px_4 & - & 4x_5 & = & -4 \\ 2x_1 & + & 3x_2 & - & 5x_3 & & & & & = & 2 \end{array}$$

where p and q are arbitrary constants.

(a) For what values of p and q does this system have a unique solution, several solutions and no solution, respectively?

(b) Find the solutions for $p = 0$ and $q = \frac{2}{3}$.

4. A firm wants to produce two lines of product, A and B. The two needed raw materials R_1 and R_2 are available at 100 tons R_1 and at 120 tons R_2. The production equipment can be used for 150 hours.

	Product A	Product B
Raw material R_1 needed per unit of product	1 ton	2 tons
Raw material R_2 needed per unit of product	2 tons	2 tons
Production time needed per unit of product	3 hours	2 hours

(a) Find all feasible solutions for the problem graphically.

(b) Find one basic feasible solution by calculation representing useful output combinations.

5. Determine all local extreme points of

$$f(x_1, x_2, x_3) = e^{x_1^3 - 3x_1} - x_2^2 + x_2x_3 - x_3^2 + 3x_3.$$

6. After placing a new product on the market the sales y will grow up. It can be assumed that the rate of change ϱ of y as a function of time t follows the function $\varrho_y(t) = 3\sqrt{2t+1}$.

(a) Find the sales function $y(t)$ by solving the resulting differential equation.

(b) Find the special sales function when $y = e$ for $t = 0$.