Original

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Examination in Mathematics B

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Working time: 120 minutes

The derivation of the results must be given clearly. The statement of the result only is not sufficient.

Tools:

- pocket calculator
- printed collection of formulas
- lecture notes

Problems:

1. A client wants to build his own house for 220,000 DM. He has 75,000 DM of his own capital and he wants to be free of depts after 20 years. To finance his project, the client makes a contract with a building and loan association with the following conditions:
   At the beginning on January 1,1999, the client makes a deposit of 75,000 DM in a savings agreement for which he gets 3% p. a. At the same time he gets a loan of 220,000 DM for which he only has to pay interest of 7.5% per annum at the end of the year. At the end of that year during which the only payment of 75,000 DM plus its interest reach at least 40% of the whole contract sum of 220,000 DM, the client will get 220,000 DM from the savings agreement und repay the first loan. From that time the client has to amortize the present value of the loan by payments at the end of the year and interest at 7.5% p.a. compounded annually.
   Find the total payment and the time when the first loan can be repaid.

2. Given is the function

$$y = f(x) = (\ln x)^3 - 3\ln x \quad .$$

   (a) Find domain, zeros and the limits at the boundaries of the domain.

   (b) Determine the extreme points of the function $f(x)$.

   (c) Verify that the function $f(x)$ changes its elasticity in the interval $[\sqrt{e}, e]$.

3. Find the local extrema of the function

$$z = 2x^3 + 3xy + 3x^2 + \frac{1}{2}y^2 - 9x - 2y + 5 \quad .$$

4. Given is an ellipsoid $x^2 + \frac{y^2}{4} + \frac{z^2}{9} = 1$ with its centre as the origin of rectangular Cartesian co-ordinates x, y, z. This ellipsoid may contain rectangular parallelepipedons (cuboids) with edges parallel to the co-ordinate axes. When the point of contact between the ellipsoid and the cuboid in the 1. octant is (x, y, z) the volume of the cuboid can be given as $V = 8xyz$. Find length, breadth and hight of the cuboid inside the ellipsoid so that its volume becomes a maximum. Use the Lagrange-multiplier method. Determine the bordered Hessian for this problem. It is not required to check the local suffient conditions.

5. Determine the values of the following integrals:

   (a)

$$\int_0^{\pi^2} \sin\sqrt{x}\, dx$$

   (b)

$$\int_0^{\infty} (x^2 + 1)e^{-x}\, dx$$