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<u>Examination in Mathematics II</u>

(13.02.2008)

Working time: 120 minutes

The derivation of the results must be given clearly. The statement of the result only is not sufficient.

Tools:

- pocket calculator
- printed collection of formulas
- **either** two individually prepared double-sided sheets of paper (write '2' on cover sheet) **or** textbook 'Mathematics of Economics and Business (write 'B' on cover sheet)

It is not allowed to use mobile phones.

Distribution of points obtainable for the problems:

problem	1	2	3	4	5	6	sum
points	7	8	9	6	12	8	50

Problems:

1. Given are the vectors

$$\mathbf{a} = \begin{pmatrix} 2 \\ 0 \\ -1 \end{pmatrix}, \qquad \mathbf{b} = \begin{pmatrix} 1 \\ -3 \\ 1 \end{pmatrix} \qquad \text{and} \qquad \mathbf{c} = \begin{pmatrix} -1 \\ -1 \\ 1 \end{pmatrix}.$$

   (a) Do the vectors $\mathbf{a}$, $\mathbf{b}$ and $\mathbf{c}$ constitute a basis in $\mathbb{R}^3$?

   (b) For which values $\mu \in \mathbb{R}$ can vector

$$\mathbf{d} = \begin{pmatrix} \mu \\ 1 \\ 2 \end{pmatrix}$$

   be written as a linear combination of the vectors $\mathbf{a}, \mathbf{b}$ and $\mathbf{c}$?

   (c) Give for the value of μ found in (b) vector $\mathbf{d}$ as a linear combination of vectors $\mathbf{a}, \mathbf{b}$ and $\mathbf{c}$.

2. A linear mapping A which assigns to any

$$x = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} \in \mathbb{R}^3 \qquad \text{uniquely a} \qquad y = \begin{pmatrix} y_1 \\ y_2 \\ y_3 \end{pmatrix} \in \mathbb{R}^3$$

   is described by

$$\begin{pmatrix} y_1 \\ y_2 \\ y_3 \end{pmatrix} = \begin{pmatrix} \lambda & 1 & 2 \\ -2 & 0 & 2 \\ 1 & 1 & 1 \end{pmatrix} \cdot \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$$

   (a) Specify the inverse mapping for $\lambda = -1$.

   (b) For which value $\lambda \in \mathbb{R}$ does an inverse mapping **not** exist?

3. Given is the system of linear inequalities

$$\begin{array}{rcrcr} -x_1 & + & x_2 & \geq & -2 \\ 6x_1 & + & 8x_2 & \leq & 48 \\ -4x_1 & - & 3x_2 & \leq & -12 \end{array}$$

$$x_1, x_2 \geq 0$$

(a) Determine the extreme points of the given system of linear inequalities exactly and give the general solution of this system.

(b) Consider in addition to the above system the objective function

$$f(x_1, x_2) = 2x_1 - x_2 \to \max!$$

Determine the starting tableau for applying the simplex algorithm.

4. For which values of variable z is the following matrix H negative definite:

$$H = \begin{pmatrix} z & 1 & -1 & 2 \\ 1 & -2 & 1 & 0 \\ -1 & 1 & -1 & 0 \\ 2 & 0 & 0 & -2 \end{pmatrix}.$$

5. Given is the function $f : \mathbb{R}^3 \to \mathbb{R}$ with

$$f(x, y, z) = 2e^{x^3-3x} - y^2 - z^2 + yz + 3z + 250$$

(a) Determine all local extreme points of function f.

(b) Determine the directional derivative of function f at the point $P = (1, 1, 1)$ in the direction given by $\mathbf{r}^T = (1, 2, 2)$.

6. The elasticity of a demand function D with $D(p) > 0$ (p denotes the price) is given by

$$\epsilon_D(p) = -2\,\frac{p^2}{p^2+1} - p.$$

(a) Determine all functions D having the above elasticity.

(b) Determine among all solutions the particular demand function with $D(1) = 8$.