Original

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Examination in Mathematics II

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Working time: 120 minutes

The derivation of the results must be given clearly. The statement of the result only is not sufficient.

Tools:

- pocket calculator
- printed collection of formulas
- script "Mathematics for Students of Economics and Management"

It is not allowed to use mobile phones.

Problems:

1. Find the solution of the following equation with matrices:

$$A \cdot X - A = I + X$$

where I is the identity matrix and $A = \begin{pmatrix} 1-a & 0 & 0 \\ a & 1-a & 0 \\ 0 & 0 & 2-a \end{pmatrix}$.

For which value a does a solution not exist?

2. The demand function for a commodity takes the form

$$q(p) = a + bp + \frac{c}{p}$$

for some constants a, b, c.

(a) When $p = 1$, the quantity demanded is 60, when $p = 2$, it is 40, and when $p = 4$, it is 15. Find the constants a, b, c.

(b) Let $p = 1$ with $q = 60$ and $p = 2$ with $q = 40$. Find an interval for c such that a and b are positive.

3. Each of two firms A_1 und A_2 offers a single product. The relations between the outputs x_1, x_2 and the prices p_1, p_2 are the following:

$$x_1 = 100 - 2p_1 - p_2,$$
$$x_2 = 120 - p_1 - 3p_2.$$

The total costs are given by

$$c_1(x_1) = 120 + 2x_1,$$
$$c_2(x_2) = 120 + 2x_2.$$

(a) Find the profit functions g_1, g_2 for both firms and the sum of the profit functions $g = g_1 + g_2$ as functions of p_1, p_2. Remember that profit is the difference of total-revenue function and total-cost function where total revenue is $x_i p_i$ $(i = 1, 2)$.

(b) Determine p_1, p_2 such that the total profit g is maximized. What is the maximum profit?

(c) After some troubles between both firmes, A_2 fixes the price $p_2 = 16$. What price p_1 does ensure that profit g_1 is maximized? What is the effect on the total profit function g?

4. Use the Lagrange-multiplier method for the decision whether

$$f(x, y, z) = 2x + y + z,$$

subject to

$$g_1(x, y, z) = x^2 + y^2 - 8 = 0$$

and

$$g_2(x, y, z) = 2x + z - 1 = 0$$

has a maximum at

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 2\sqrt{2} \\ 1 \end{pmatrix}.$$

5. Find respectively evaluate the following integrals:

(a) $\displaystyle\int \frac{dx}{x \ln x}$

(b) $\displaystyle\int_0^{\pi} e^{-x} \cdot \sin 2x \, dx$

(c) $\displaystyle\int_1^{\infty} \frac{dx}{\sqrt{x}(1+x)}$

6. The elasticity of the function $y(x)$ is given by

$$\varepsilon_y(x) = 2x + 1\,.$$

Find the function $y(x)$ by using the definition of elasticity

$$\varepsilon_y(x) = x \cdot \frac{y'(x)}{y(x)} \text{ (differential equation).}$$