OTTO VON GUERICKE UNIVERSITÄT MAGDEBURG

WW FACULTY OF ECONOMICS AND MANAGEMENT

Microeconomic Analysis

(20024)

Examination Winter Semester 2012/13

Examiner:	Prof. Dr. Andreas Knabe
Date:	09.02.2013
The following aids may be used:	Non-programmable pocket calculators Bilingual English language dictionaries without individual entries or marking
Time:	120 minutes

Including the front page this exam contains 3 pages with 3 questions. The total amount of points to be obtained is 45. When a written explanation is asked for, please answer in short, but complete sentences and **not** just in catchwords. Remember that you should carefully explain all elements when providing graphical illustrations.

Good luck!

Question 1: Consumer Theory (15 points)

a) Which assumptions need to be fulfilled to form a continuous real valued function, $u: \mathbb{R}^n_+ \to \mathbb{R}$, which represents a preference relation $\succsim$. Explain briefly in your own words all of the assumptions.

4 points

Assume an individual possesses the following utility function: $u(x_1, x_2) = \ln x_1 + \ln x_2$.

b) Derive the individual's Marshallian demand, its indirect utility function and expenditure function. From this, derive its Hicksian demand functions using Shephard's Lemma.

8 points

c) Prove that if $u(x)$ is continuous and strictly increasing on $\mathbb{R}^n_+$, then the indirect utility function $v(\boldsymbol{p}, y)$ is homogeneous of degree zero in $(\boldsymbol{p}, y)$.

3 points

Question 2: Partial Equilibrium (15 points)

Let $u(x_1, x_2) = ({x_1}^{\rho} + {x_2}^{\rho})^{1/\rho}$ be an individual's utility function. The corresponding indirect utility function is given by $v(p_1, p_2, y) = y({p_1}^r + {p_2}^r)^{-1/r}$, with $r = \frac{\rho}{\rho - 1}$.

a) Derive the individual's Marshallian demands using Roy's identity.

3 points

Now assume $p = 0.5$

b) In an initial period following coefficients are known: $p_1^1 = 1$, $p_2^1 = 1$, $y^1 = 100$, where the superscript indicates time.
In a second period the price of good two increases from $p_2^1 = 1$ to $p_2^2 = 2$ (while the price of good two remains unaltered $p_1^1 = p_1^2 = 1$). Calculate the compensating variation and the equivalent variation caused by the increase of the price of good 2.

6 points

c) Write down the formula to calculate the change in consumer surplus (you do not need to actually compute the change in consumer surplus). Should it be larger or smaller than the compensating variation?

2 points

d) Give an intuition on why the compensating variation is larger than the equivalent variation. Do this using economic intuition and graphical analysis.

4 points

Question 3: Theory of the Firm/Partial Equilibrium (15 points)

a) Prove that if $f: \mathbb{R}^n_+ \to \mathbb{R}_+$ is a production function satisfying continuity, strong monotonicity and strict concavity (and $f(\mathbf{0}) = 0$), then the profit function $\pi(p, \boldsymbol{w})$ is convex in $(p, \boldsymbol{w})$.

3 points

b) Prove that if $f: \mathbb{R}^n_+ \to \mathbb{R}_+$ is a strictly concave production function satisfying continuity, strong monotonicity and strict quasiconcavity (and $f(\mathbf{0}) = 0$) and assuming its associated profit function, $\pi(\boldsymbol{p}, y)$, is twice continuously differentiable, then, for all $p > 0$ and $\boldsymbol{w} \gg \mathbf{0}$ where it is well defined the following holds:

$$\frac{\partial y(p,w)}{\partial p} \geq 0$$

$$\frac{\partial x_i(p,w)}{\partial w_i} \leq 0, \text{ for all } i = 1, \ldots, n.$$

4 points

c) In a market with two firms, the inverse demand is given by $p(q_1, q_2) = a - bq_1 - bq_2,\ with\ a, b > 0$. Firm 1 has the following cost function $C_1(q_1) = \alpha q_1$, the cost function for firm 2 is $C_2(q_2) = q_2$. What is the price in equilibrium if both firms decide simultaneously on quantities supplied? What is the price in equilibrium if firm 1 decides first on its quantity and then firm 2?

8 points