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Examination – Statistics I

1	2	3	4	5	Σ

Please note the following:

- The exam consists of 5 problems with a total of 36 questions;
- For each question there are several possible answers. Exactly one of them is correct.
- You get 1 point for every correct answer.
- You get $-1/2$ points for every incorrect answer.
- You get 0 points for not answering a question.
- You can reach a maximum of **36 points**. For passing the exam a total of **17 points** is sufficient.
- You are allowed to use: Pocket calculators, text books, mathematical and/or statistical tables, manuscripts and notes from lectures and/or exercises.

Problem 1.

Assume that X_1 and X_2 are real-valued continuous random variables. Then

a) $\text{Var}(X_1 + X_2) = \text{Var}(X_1) + \text{Var}(X_2)$
- ☐ Correct Statement
- ☐ Wrong Statement

b) $E(X_1 \cdot X_2) \leq E(X_1) \cdot E(X_1)$
- ☐ Correct Statement
- ☐ Wrong Statement

c) $P(X_1 \geq 10, X_2 \geq 10) = P(X_1 \geq 10) + P(X_2 \geq 10) - P(\max(X_1, X_2) \geq 10)$
- ☐ Correct Statement
- ☐ Wrong Statement

d) $P(X_1 \geq 10) = P(X_1 > 10)$
- ☐ Correct Statement
- ☐ Wrong Statement

e) $X_1 + X_2$ is also a continuous random variable
- ☐ Correct Statement
- ☐ Wrong Statement

Problem 2.

The time (in minutes) needed to serve a single costumer at a supermarket check-out counter is assumed to follow an exponential distribution. The average service time is 2 minutes.

a) The parameter of this exponential distribution is given by

☐ 120

☐ 2

☐ 1/2

b) The distribution function of this random service time is given by

☐ $F(x) = \begin{cases} 1 - \exp(-x/2) & \text{if } x \geq 0 \\ 0 & \text{otherwise} \end{cases}$

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☐ $F(x) = 1 - \exp(-2x),\ x \in \mathbb{R}$

c) The probability that the service time does not exceed 2 minutes is approximately

☐ 0.63

☐ 0.51

☐ 0.47

d) The conditional probability that the service will need at least 2 more minutes, given that it already needed at least 3 minutes, is approximately

☐ 0.15

☐ 0.37

☐ 0.28

e) If the service times of 4 single costumers are independent and follow the same exponential distribution (as above), then the probability that at least one the 4 custumers needs more than $2 \cdot \ln 2$ minutes to be served is exactly given by

☐ 0.2135

☐ 0.9852

☐ 0.3439

f) and the variance of the sum of these 4 service times is equal to

☐ 2

☐ 16

☐ 4

Problem 3.

The joint distribution of a discrete bivariate random vector (X_1, X_2) is given by the following table, which contains the probabilities $P(X_1 = x_1, X_2 = x_2)$ for $x_1 = 0, 2, 4$ and $x_2 = 0, 1, 2$:

	0	1	2
0	1/12	0	0
2	0	1/3	1/3
4	0	0	1/4

a) The probability $P(X_1 = 2)$ is given by

- ☐ 3/4
- ☐ 2/3
- ☐ 1/12

b) The conditional probability $P(X_2 = 2 \mid X_1 = 2)$ is given by

- ☐ 1/3
- ☐ 2/3
- ☐ 1/2

c) The expected value $E(X_1)$ is given by

- ☐ 1
- ☐ 1/12
- ☐ 7/3

d) The variance $\text{Var}(X_2)$ is given by

- ☐ 5/12
- ☐ 1/9
- ☐ 2/3

e) The expected value $E(X_1 \cdot X_2)$ is given by

- ☐ 5/2
- ☐ 4
- ☐ 2

f) The covariance $\text{Cov}(X_1, X_2)$ is given by

- ☐ 1/4
- ☐ 1/2
- ☐ 1

g) The random variables X_1 and X_2 are

- ☐ positive correlated
- ☐ independent
- ☐ uncorrelated, but not independent

h) The variance $\mathrm{Var}(X_1 + X_2)$ satisfies

- ☐ $\mathrm{Var}(X_1 + X_2) = \mathrm{Var}(X_1) + \mathrm{Var}(X_2)$
- ☐ $\mathrm{Var}(X_1 + X_2) > \mathrm{Var}(X_1) + \mathrm{Var}(X_2)$
- ☐ $\mathrm{Var}(X_1 + X_2) = 2 \cdot \mathrm{Var}(X_1)$