Examination: **Statistics I** July 26, 2004

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Please note the following:

- The exam consists of 6 problems for solution; the points you can get for each problem are given in brackets next to the number of the problem. You do not have to solve the individual problems completely, partial solutions are also possible. For problems 2-6 it is not enough, however, to state the result only, but you should clearly display your approach and way to solution.
- You can achieve a total of 45 points. For passing the exam you have to achieve a total of (at least) **20 points** from all problems.
- You are allowed to use: Pocket calculators, text books, mathematical and/or statistical tables, manuscripts and notes from lectures and/or exercises.

Problem 1. *(5 points)*
The following statements are TRUE or FALSE. So just answer TRUE or FALSE without giving any explanation.

(a) If X_1 and X_2 are independent random variables with variances σ_1^2 and σ_2^2, respectively, then

$$\mathrm{Var}(X_1 - X_2) = \sigma_1^2 - \sigma_2^2.$$

(b) If X is a random variable with expectation μ then $E(X^2) = \mu^2$.

(c) A probability density is a nonnegative increasing function.

(d) The covariance of two random variables is a number between -1 and 1.

(e) If $X \sim N(2, 4)$ then $1/2 \cdot X - 1 \sim N(0, 1)$.

Problem 2. *(7 points)*
The following table shows the scores obtained by 8 persons in a psychological test.

person	1	2	3	4	5	6	7	8
score	0	4	2	5	2	0	1	10

(a) Determine a 0.3-quantile and a median of this data.

(b) Compute the empirical distribution function and draw it.

(c) The sample variance and the sample mean of the score data are given by 78/7 and 3, respectively. Use this information to compute the arithmetic mean of the squared scores.

Problem 3. *(10 points)*
The processing time (in minutes) of a certain job on a pc is modelled by a normal distribution with mean 0.5 and variance 0.04. If several jobs of that type are carried out, one after the other, the corresponding processing times are assumed to be independent.

(a) If 5 jobs have to be done, what is the probability that at least one of them needs more than 30 seconds processing time?

(b) If 4 jobs have to be done, what is the probability that the total processing time for the first three of them is shorter than the processing time for the last job?

(c) Can one guarantee that with probability at least 90% a total of 100 jobs can be carried out within one hour?

Problem 4. *(10 points)*
In a lottery game two numbers are drawn, each of them beeing either 1 or -1. The game is modelled by a bivariate random variable (X, Y) with joint probabilities $P(X = x, Y = y)$ as stated in the table below.

x \ y	-1	1
-1	1/4	1/8
1	1/2	1/8

(a) Are the random variables X and Y independent?

(b) Compute the correlation of X and Y.

(c) You receive 10 Euro if the product of the two numbers is positive. Otherwise you have to pay 7 Euro. If you play 8 times, what is your expected gain?

Problem 5. *(7 points)*
You have had a nice picknic at a place near to a small forrest. Back to your car you realize that your dog is missing. From experience you know that

- with 20% chance your dog is still around the picknic place,
- with 70% chance your dog is in the forrest.
- with 10% chance your dog is elsewhere.

You decide to return and try to find the dog either at the picknic place or in the forrest. You do not search it anywhere else. If the dog is at the picknic place you will find it with 90% chance. If the dog is in the forrest you only have a chance of 50% to find it.

(a) Compute the probability that you will find your dog.

(b) Given that you found the dog what is the probability that this has happened at the picknic place?

Problem 6. *(6 points)*
The number of typos per page of a book with 1000 pages is assumed to follow a Poisson distribution with parameter 0.1. Under the assumption of independence estimate the probability that the total number of typos in the book lies between 80 and 120

(a) using the Central Limit Theorem,

(b) using the Tchebychev inequality.