Orginal

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Statistics II — Examination

Please note the following

- The exams consists of 5 problems for solution; for each problem you can get at most 10 points. You do not have to solve the individual problems completely, partial solutions are also possible. It is not enough, however, to state the result only, but you should clearly display your approach and way to solution.
- For passing the exams you have to achieve a total of (at least) **14 points** from all problems.
- You are allowed to use: Pocket calculators, text books, mathematical and statistical tables, manuscripts and notes from the lectures and exercises.

Good luck !

Problem 1 (10 pts)
The prices (in DM) for 1 liter unleaded fuel at the gasstations in a city are assumed to be independent and normally distributed with unknown mean μ and (known) standard deviation $\sigma_0 = 0.02$.

Two independent samples of fuel prices are available. It is desired to combine the two individual sample means, $\overline{x}$ and $\overline{y}$, say, to a single estimate of the mean price μ per liter. The (relevant) sample summaries are as follows.

	sample 1 x	sample 2 y
sample size	$n_x = 10$	$n_y = 6$
sample mean	$\overline{x} = 1.759$	$\overline{y} = 1.779$

(a) Show that both estimators, $\widehat{\mu}_1 = \frac{1}{2}(\overline{x} + \overline{y})$ and $\widehat{\mu}_2 = \frac{1}{n_x + n_y}(n_x\overline{x} + n_y\overline{y})$, are unbiased estimators of μ; compute the two estimates based on $\hat{\mu}_1$ and $\hat{\mu}_2$, respectively.

(b) Which if the two estimators from part (a) is more efficient?

Problem 2 (10 pts)
A sample of credit card purchase amounts (in USD) is gathered from two independent stores; the data are shown in the table.

store 1			store 2	
21.9	19.5	54.7	35.3	67.3
22.5	25.6	73.3	76.3	32.1

Assume that the purchase amounts for the individual stores are independent and come from normal populations with the same standard deviation σ.

Construct a (symmetric, twosided) 95%–confidence interval for the difference between the two mean purchase amounts of the stores.

Problem 3 (10 pts)
A random sample of $n = 51$ saving accounts in a local community shows a mean percentage increase over the last year of $\overline{x} = 7.2$ and a standard deviation of $s = 5.6$, (both in percent). The percentage increases are assumed to be independent and normally distributed.

(a) Test at the (approximate) level $\alpha = 0.05$ the null hypothesis that the mean percentage increase μ is at least 3 against the alternative that it is less than 3.

(b) Is there enough evidence in the sample that the population standard deviation exceeds the value 4? Again, use the level $\alpha = 0.05$.

Problem 4 (10 pts)
$n = 600$ books were borrowed from a public library during the five days of a particular week. The number of books borrowed at the individual days are given below.

Day	1 (Monday)	2 (Tuesday)	3 (Wednesday)	4 (Thursday)	5 (Friday)
Books	125	100	110	130	135

(a) Is there enough evidence for rejecting the hypothesis that the number of books borrowed does not depend on the day, (i.e., for rejecting the hypothesis that the probability of borrowing a book is 0.2 for each of the five days)? Use the level $\alpha = 0.1$.

(b) Without doing the computations, is the corresponding p-value smaller or larger than 0.1?

Problem 5 (10 pts)
A test is given to each new employee at a particular company. This test is divided into two parts. One part concerns technical knowledge, the other tests leadership qualities. The scores obtained by $n = 7$ new employees are shown in the table.

	employee						
	1	2	3	4	5	6	7
technical knowledge	33	42	26	29	30	45	22
leadership ability	41	39	35	26	37	38	31

(a) Find the Spearman rank correlation coefficient for these data.

(b) Test at the level $\alpha = 0.05$ the null hypothesis of no correlation against the alternative of a positive correlation.