Otto–von–Guericke–Universität Magdeburg **February 14, 2003**

Examination: **Statistics II (Bachelor in Management/Economics)** summer 03

Examiner: Professor Dr. R. Schwabe

Please note the following:

- The exam consists of 5 problems for solution; the points you can get for each problem are given in brackets next to the number of the problem. If you are not able to solve the individual problems completely, partial solutions are also accepted. For problems 2-5 it is not enough, however, to state the result only, but you should clearly display your approach and your way to the solution. Also, *conclusions* are to be drawn where required.
- For passing the exam you have to achieve a total of (at least) **16 points** out of 37 attainable.
- You are allowed to use: Pocket calculators, text books, mathematical and/or statistical tables, manuscripts and notes from lectures and/or exercises.
- **Good luck !**

Problem 1. *(7 points)*
In the following $X_1, \ldots, X_n$ is an independent sample of size n from a normal distribution with known mean μ and unknown variance σ^2. Are the following statements correct or wrong? (Just answer *Yes* or *No.*)

(a) The larger the sample size n, the larger is the sample mean.

(b) The larger the sample size n, the smaller is the mean of the sample variance.

(c) $\frac{1}{n}\sum_{i=1}^{n}(X_i - \mu)^2$ is an unbiased estimator for the population variance σ^2.

(d) $\sqrt{n}\,\frac{\overline{X} - \mu}{\sigma}$ has a standard normal distribution.

(e) Let Y be a continuous random variable. If 0.2 is a 0.9-quantile of the distribution of Y, then $P(Y > 0.2) = 0.1$.

(f) When a significance test rejects the null hypothesis at significance level $\alpha = 0.03$, then the p-value of that test is greater than 0.03.

(g) When the null hypothesis is rejected by a significance test, it means that there is sufficient evidence to conclude that the alternative hypothesis is true.

Problem 2. *(6 points)*
The salary of managers was asked in two different samples X and Y, say. The salaries are assumed to be independent and normally distributed with common mean μ and common standard deviation σ.
It is desired to combine the two individual sample means $\bar{x}$ and $\bar{y}$ in order to get a single estimate of the "typical" salary μ. The relevant informations from the samples are as follows.

	sample X	sample Y
sample size	$n_x = 10$	$n_y = 20$
sample mean	$\bar{x} = 201\,000$	$\bar{y} = 198\,500$

Two alternative estimators, $\hat{\mu}_1 = \frac{1}{2}(\bar{x} + \bar{y})$ and $\hat{\mu}_2 = \frac{1}{(n_x + n_y)}(n_x\bar{x} + n_y\bar{y})$, are proposed.

(a) Show that $\hat{\mu}_1$ and $\hat{\mu}_2$ are both unbiased estimators for μ and compute the corresponding values of the estimates.

(b) Calculate the variances $\mathrm{Var}(\hat{\mu}_1)$ and $\mathrm{Var}(\hat{\mu}_2)$ of the estimators $\hat{\mu}_1$ and $\hat{\mu}_2$, respectively. Which estimator is more efficient ?

Problem 3. *(6 points)*
To determine the impact of a company's research effort on its financial performance experts were asked to judge on these attributes. For a sample of 117 companies the following counts were observed.

		research effort		
		low	high	total sum
financial performance:	low	15	23	38
	high	18	61	79
	total sum	33	84	117

(a) Is there any evidence at the 10 % level that the financial performance is **not** independent of the research effort?
(b) Is it equally likely that the financial performance is rated high or low by the experts ($\alpha = 0.05$)?

Problem 4. *(10 points)*
Two airlines shall be compared with respect to their delays. Delays are considered to be normally disributed.

A sample of 41 randomly chosen flights of airline 1 showed a mean delay [in minutes] of 13.5 with standard deviation 4.4.

A sample of 31 randomly chosen flights of airline 2 showed a mean delay of 15.6 with standard deviation 4.2.

Make suitable assumptions (write them down!) and answer the following questions:

(a) "Does one of the airlines have a greater average delay than the other ?"
Construct a 95% confidence interval to answer the question.
(b) "Does airline 1 have a significantly smaller average delay than airline 2 ?"
Perform a significance test to answer the question. Use significance level $\alpha = 0.05$.
(c) "Do the variances of the two populations differ significantly ?"
Perform a significance test at level $\alpha = 0.05$ to answer the question.
(d) 7 out of the 41 examined flights of airline 1 were not delayed. Perform a significance test at level $\alpha = 0.05$ in order to answer the question: "Is there enough evidence to claim that more than 75% of the flights of airline 1 are delayed?"

Problem 5. *(8 points)*
Usually it might be assumed that the profit y of a company depends on its sales x. The following data show the sales and profits [in millions of US-$] for ten small computer companies (extracted from the April 23, 1990 edition of *Fortune* magazine).

Sales ($M)	Profits ($M)	Sales ($M)	Profits ($M)	Sales ($M)	Profits ($M)
855.1	31.0	2 153.7	153.0	709.3	41.4
784.7	89.0	873.6	79.5	990.5	20.9
1 382.3	0.3	1 014.0	47.7	1 769.2	60.8
1 643.9	118.3				

(a) Compute the correlation coefficient. (Hint: The sample standard deviations and the sample covariance are given by $s_x = 491.9$ M\$, $s_y = 46.6$ M\$, and $s_{xy} = 12\,734.1$, respectively.)
(b) Calculate the regression coefficients for a regression line and graph it on the attached scatter plot.
(c) Interpret the estimated values $\hat{\beta}$ and $\hat{\alpha}$ for the slope and the intercept, respectively.
(d) Is there enough evidence that there is a dependence between sales and profits ($\alpha = 0.05$)?