Examination Statistics II

February 25, 2005

Otto-von-Guericke-Universität Magdeburg
Fakultät für Mathematik
PD Dr. T. Müller-Gronbach

Please note the following:

- The exam consists of 5 problems; the credit points you can get for each problem are given in brackets next to the number of the problem. You do not have to solve the individual problems completely, partial solutions are also possible. Except for Problem 1, it is not enough to state the result only, but you should clearly display your approach and way to solution.
- You can reach a maximum of **50 points**. For passing the exam a total of **22 points** is sufficient.
- You are allowed to use: Pocket calculators, text books, mathematical and/or statistical tables, manuscripts and notes from lectures and/or exercises.

Problem 1 *(5 points)*
The following statements are TRUE or FALSE. Just answer TRUE or FALSE without giving any explanation.

(a) A maximum-likelihood estimator is an unbiased estimator.

(b) The bias of an estimator may take negative values.

(c) If φ is a 0.05-level test for H_0 vs. H_1, then φ is also a 0.07-level test for H_0 vs. H_1.

(d) The length of a confidence interval does not depend on the data.

(e) If a 0.01-level test for H_0 versus H_1 rejects H_0, then the alternative H_1 is true with probability at least 0.99.

Problem 2 *(12 points)*
A certain delay time is modelled by a random variable X with density function

$$f_\theta(x) = \begin{cases} 2x/\theta^2 & \text{if } 0 \le x \le \theta \\ 0 & \text{otherwise,} \end{cases}$$

where $\theta \in [1, 2]$ is an unknown parameter.

(a) Show that $E(X) = 2\theta/3$ and $E(X^2) = \theta^2/2$.

Consider a random sample $X_1, \dots, X_n$ i.i.d. as X and the estimator $\hat{\theta} = 3/2 \cdot \overline{X}$.

(b) Show that $\hat{\theta}$ is an unbiased estimator of θ and compute its variance.

(c) Compute the minimum sample size n necessary to obtain a mean squared error of $\hat{\theta}$ below 0.01 for all possible values of θ.

(d) Assume that $n = 3$ and that the data $x_1 = 1$, $x_2 = 1.3$, $x_3 = 1.7$ has been observed. Compute the resulting maximum-likelihood estimate of θ. Compare with the estimate obtained by $\hat{\theta}$.

Problem 3 *(14 points)*
The following table shows the copper levels $x_1, \dots, x_{10}$ (in mg/l) for a random sample of 10 water probes from the same drinking water resource.

0.508	0.279	0.320	0.904	0.221
0.283	0.475	0.130	0.220	0.743

The sample mean and the sample standard deviation of the above data are approximately given by $\overline{x} = 0.41$ and $s = 0.25$.

Assume a normally distributed copper level.

(a) Compute a 95%-confidence interval for the mean copper level.

(b) Compute 90%-confidence intervals for the variance and for the standard deviation of the copper level.

The next table shows the ordered data $x_{(i)}$ and the corresponding values $F(x_{(i)})$ for the distribution function F of $\mathrm{N}(0.4, 0.06)$.

i	1	2	3	4	5	6	7	8	9	10
$x_{(i)}$	0.130	0.220	0.221	0.279	0.283	0.320	0.475	0.508	0.743	0.904
$F(x_{(i)})$	0.140	0.236	0.237	0.314	0.320	0.374	0.618	0.667	0.915	0.978

(d) Is there significant evidence, at level 0.05, that the copper level does not follow a normal distribution with mean 0.4 and variance 0.06? State the testing problem as well as the testing procedure you are using.

Problem 4 *(9 points)*
A study in the Annals of Tourism Research (1992) investigates the relationship of retirement status to the length of stay of a trip. A sample of 703 travelers were asked how long they stayed on a typical trip. The results are shown in the following table.

	Retirement Status		
Length of typical trip	Preretirement	Postretirement	Total
4-7 nights	247	172	419
8-13 nights	82	67	149
14 or more nights nights	51	84	135
Total	380	323	703

Is there significant evidence, at level 0.05, that retirement status and length of trip are dependent? State the testing problem and justify the testing procedure you are using.

Problem 5 *(10 points)*
In a study of the feeding behaviour of blackbream fish, zoologists recorded the number of aggressive strikes of two blackbream fish in an aquarium during the 10-minute period following the addition of food. The following table lists the weekly number of strikes and age of the fish (in days).

Week	Number of Strikes	Age
1	85	120
2	63	136
3	34	150
4	39	155
5	58	162
6	35	160
7	57	178
8	12	184
9	15	190

(a) Draw a scatterplot of the age-strike data.

The sample standard deviations are 23.66 and 22.87 for the strikes and the age, respectively. The empirical correlation coefficient is -0.79. The sample means are 44.22 and 159.44 for the strikes and the age, respectively.

Use linear regression to model the dependence of the number of strikes on the age.

(b) Compute the least squares estimates of the intercept and the slope of the regression line. Draw this line into the scatterplot.

(c) Do the data provide significant evidence, at level 0.01, that on the average the number of strikes decreases with increasing age? State the model you are using. State the testing problem and the testing procedure.