FACULTY OF ECONOMICS AND MANAGEMENT

Advanced Labor Economics

(20628)

Examination Summer Semester 2013

Examiner:	Prof. Dr. Andreas Knabe
Date:	29.07.2013
The following aids may be used:	Non-programmable pocket calculators.
	Bilingual English language dictionaries without individual entries or marking.
Time:	60 minutes.

Including the front page this exam contains 3 pages with 2 questions, first question containing 3 subquestions and second question containing 6 subquestions. The total amount of points to be obtained is 60. When a written explanation is asked for, please answer in short, but complete sentences and **not** just in catchwords. Remember that you should carefully explain all elements when providing graphical illustrations!

Good Luck!

Question 1 (24 points): Education, Training and Life-Cycle Earnings

In the following model, the individual decides at each point in time $t \in [0; T]$ about the share of time $s(t) \in [0; 1]$ to invest in education. The discount factor is r, the amount of human capital an individual possesses is $h(t)$. The wage income an individual receives when working is $A[1 - s(t)]h(t)$. The aim of the individual is to maximize the discounted lifetime income:

$$\Omega = \int_0^T A[1 - s(t)]h(t)e^{-rt}dt,$$

subject to the law of motion (δ is the rate of depreciation of knowledge; $g' > 0, g'' < 0$):

$$\dot{h}(t) = \theta g[s(t)h(t)] - \delta h(t)$$

a) Show the present-value *Hamiltonian* H of the intertemporal optimization problem. Furthermore, derive the optimality conditions of the *control variable* and the *state variable*. **(10 points)**

Keep in mind the general rules of derivation for some *control variable* $C(t)$ and some *state variable* $K(t)$:

$$\frac{\partial H}{\partial C(t)} = 0$$
$$\frac{\partial H}{\partial K(t)} = -\dot{\lambda}(t)$$

b) Combining the two optimality conditions derived in a) yields a differential equation of $\lambda(t)$. The solution of the differential equation is (with c being a constant of integration):

$$\lambda(t)e^{rt} = ce^{(r+\delta)t} + \frac{A}{(r + \delta)}$$

Determine the constant of integration c and the corresponding function $\lambda(t)$ which represents the marginal value of human capital at date t. Explain in words how the derived path of $\lambda(t)$ can be used to solve for the optimal path of $s(t)$ and $h(t)$. **(8 points)**

c) Depict in three different graphics **and** describe in your own words the typical paths of **(6 points)**

i) the level of human capital $h(t)$,

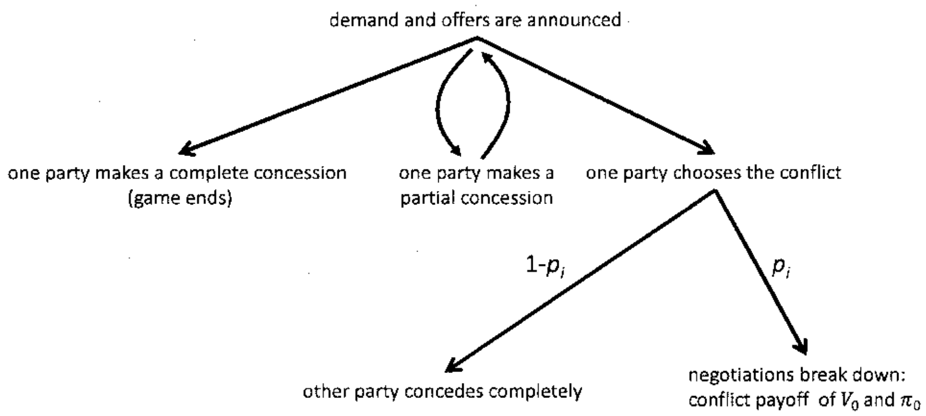
ii) the share of time spent for education $s(t)$,

iii) the instantaneous wage income (thus, the wage income at each point in time) received.

Question 2 (36 points): Bargaining Theory

The Zeuthen solution

Consider a bargaining game between a union U and a firm F over wage w. The wage demand of U is denoted by w_U and the wage offer of F by w_F. U has the utility function $V(w)$ with $V_w > 0$ and F a profit function $\pi(w)$ with $\pi_w < 0$. When negotiations fail the payoffs are V_0 and π_0. The bargaining solution is derived according to the following scheme:



a) When does U accept the offer of F? Derive the "risk threshold". **(5 points)**

b) Based on the "Zeuthen assumption", explain in your own words how the bargaining solution is derived. Derive the formal solution for the bargaining solution. **(5 points)**

The Right-to-Manage Model

Assume a wage-bill maximizing union U and a firm F. In the right-to-manage model the firm always decides about its own labor demand, but the wages are bargained over. If a worker is unemployed, he receives the "reservation wage" $\bar{w}$. The firm has a profit function $\pi(w)$ and the union aims to maximize the wage bill wL. Disagreement payoffs are 0 for the firm and $\bar{w}L$ for the union. Both parties have equal bargaining strength, i.e. we apply the symmetric Nash bargaining solution.

c) The Nash bargaining solution is given by the following maximization problem: **(5 points)**

$$\max_w \Omega = [\pi(w)]\,[L(w)(w-\bar{w})]$$

Derive the FOC and rearrange the equation so that it depends on the wage elasticity of labor demand η_w^L and the wage elasticity of profits η_w^π. Note that:

$$\eta_w^L = -\frac{\partial L}{\partial w}\frac{w}{L} \qquad \eta_w^\pi = -\frac{\partial \pi}{\partial w}\frac{w}{\pi}$$

Referring to the firm, assume a Cobb-Douglas production function with $f(L) = L^\alpha$ and a profit function $\pi = pL^\alpha - wL$.

d) What is the profit maximizing labor demand function of the firm? **(5 points)**

e) Show that the elasticities in c) take the following values: **(10 points)**

$$\eta_w^L = \frac{1}{1-\alpha} \qquad \eta_w^\pi = \frac{\alpha}{1-\alpha}$$

f) What is the bargained wage? Present it as a function of the "reservation wage" $\bar{w}$. **(6 points)**