

Examination: 20298
Advanced Methods in International Marketing
Winter Semester 2010 / 2011
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You are allowed to use a non-programmable calculator (in accordance with the instructions given by the examination office) and a translating dictionary from your native language to English (without any notes written into it). **All** of the **eight** (8) examination questions must be answered. This examination consists of **three** (3) pages and must be completed within 60 minutes.

Question 1: The two discrete random variables $X=x$ and $Y=y$, have a known joint probability distribution, $(X, Y) \sim f(x, y)$.

- a. Explain how the univariate marginal distributions are derived.
- b. By normalizing the columns of the joint probability distribution the univariate conditional distribution of the random variable $(Y|X=x)$ is derived. What does the conditional variance, $V(Y|X=x)$, tell us about the variable of interest, Y ?

Question 2: When the range of a random variable, $Y=y \{y \geq a\}$ is restricted, we say that the random variable is truncated at point a .

- a. Explain why a truncated random variable is a conditional random variable.
- b. Consider the discrete random variable $Y=y$ that takes on the six values $\{2, 4, 6, 8, 10, 12\}$ with associated discrete uniform probabilities $\{1/6, 1/6, 1/6, 1/6, 1/6, 1/6\}$. If this random variable is now truncated at the point 6 (therefore, $y > 6$), what are the truncated probabilities that $y = 4$ and that $y = 10$?

Question 3: A joint bivariate population pmf for $X=x_i$ and $Y=y_j$, $f(x_i, y_j)$:

$Y=y_j \setminus X=x_i$	3	4	5
1.2	0.045	0.080	0.045
2.4	0.120	0.120	0.120
3.6	0.085	0.300	0.085

- a. Compute $E(Y)$ and $E(Y | X=4)$
- b. Compute $C(X, Y)$ and explain whether X and Y are stochastically independent.

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Question 4: NOTE: This problem MUST be solved using the table method that was presented in this lecture course. Other statistical methods for the solution of this problem will not be accepted.

A group of basketball players are tested in order to determine whether they have been using any ability enhancing substances. The random variable $Y = y$ indicates the presence of such a substance, $y = 1$, or absence thereof, $y = 0$. Within this group of players it is known that 1.2% of the players are drug users. When the players are tested, only 92% of those who are actual drug users will test positive. The random variable $X = x$ is the test result, $x = 1$ indicates a positive test and $x = 0$ negative. The test is not perfect because only 96% of those players, who do not use any drugs at all, will receive a negative test result.

$$f(x, y)$$

		X		
		1	0	$f_2(y)$
Y	1			
	0			
	$f_1(x)$			1.0

$$(X | Y = y)$$

		X		
		1	0	
Y	1			1.0
	0			1.0

$$(Y | X = x)$$

		X	
		1	0
Y	1		
	0		
		1.0	1.0

$$\text{Bayesian Multiple Table}$$

		X	
		1	0
Y	1		
	0		

- What is the probability that a player who tests positive is not using drugs?
- What is the probability that a player that has a negative test actually is a drug user?

Question 5: Bayes' Theorem provides a logical framework for analyzing this human thought process and shows the usefulness of information.

$$\Pr(Y | X = x) = \delta \Pr(Y), \text{ where } \delta = \Pr(X | Y = y) / \Pr(X).$$

- What are the posterior probabilities in Problem 4 above.
- Explain in detail under what conditions the multiple $\delta = 1.0$.

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Question 6: Decision-makers often make a yes / no type decision. This decision can be modeled using a dichotomous random variable, $Y = y$ where $y = \{0, 1\}$. Furthermore, assume that this decision-maker has been provided with some highly relevant information, $X = x$, $(X, Y) \sim f(x, y)$.

- Assuming, $E(Y | X = x) = \alpha + \beta x$, what are the shortcomings of this model?
- In this case, the conditional random variable $(Y | X = x)$ has the Bernoulli distribution and the conditional expectation is equal to, $E(Y | X = x) = ?$

Question 7: The general structure of diffusion models is:

$$S_t = g(t) [N^* - N_t].$$

The Bass Model specifies a functional form for $g(t)$ that proves to be very useful.

- In the Bass formulation, the total sales quantity sold in time period t , S_t , is the sum of sales to two different groups of consumers. Describe the differences in the consumption behavior of these groups.
- Assume that two different products are launched on the market at the same time. One of these products has a brand name AAA (with Bass Model parameters $p = 0.12$, $q = 0.42$) and the other has a brand name BBB (with $p = 0.02$, $q = 0.42$). Which product would sell the fastest any why?

Question 8: Consider two discrete random variables $Y = y$ and $X = x$, $(X, Y) \sim f(x, y)$.

- Explain in detail how the univariate random variable Y differs from the univariate conditional random variable $(Y | X = x)$.
- Under what conditions are these two random variables the same?

**This is the End of the Examination
GOOD LUCK !**

