Examination: 1928 – Derivatives

Examiner: Prof. Dr. Peter Reichling

You are welcome to use non-programmable pocket calculators as well as English language dictionaries without any markings. This examination comprises 4 problems (on 2 pages). All of the problems are to be solved. If not required explicitly, derivations of formulas are not (!) necessary. But you have to explain all (!) the symbols that you use. **Good luck!** 

# Examination Questions (Total Number of Points: 60)

# Problem 1 (Binomial Model - 19 Points)

Consider a stock with a current price of  $100 \in$  per share. Within one period the stock price may only rise by 10% or fall by 20%. The risk-free interest rate equals 5% per period.

- (a) Show possible developments of the stock price within two periods by using a binomial tree.
- (b) Determine the price of an American
  - (i) call
  - (ii) put

option on 50 shares of this stock with an exercise price of  $90 \in$  using the two-period binomial model. Again use binomial trees to show possible developments of the option prices within the two periods.

(c) Explain the implications if the risk-free interest rate was 15 % per period (instead of 5 %).

## Problem 2 (Black-Scholes Model - 13 Points)

- (a) What kind of stochastic process does the price of the underlying asset follow within the Black-Scholes model to value European derivatives? Write down the equation of this process.
- (b) What kind of distribution of the underlying asset's rate of return follows from (a)? What are the mean and the standard deviation of the underlying asset's rate of return within a time period of length T?
- (c) What kind of distribution of future prices of the underlying asset follows from (a)?
- (d) What is the duplication principle and what does it mean within the derivation of the Black-Scholes model to value European derivatives?
- (e) What means risk-neutral valuation and where can we find it in the Black-Scholes formula to value European derivatives?
- (f) What is implied volatility?

### Problem 3 (Exchange Options - 18 Points)

- (a) Write down the Black-Scholes formula to value (European) standard put options.
- (b) Write down the Margrabe formula to value (European) options to exchange asset 2 for asset 1.
- (c) Show that the Black-Scholes formula to value European standard put options is just a special case of the Margrabe formula to value European exchange options.

## Problem 4 (Gap Options - 10 Points)

- (a) Express the payoff profile of a (European) gap put option formally and graphically.
- (b) Show how this gap put option can be created synthetically using standard (European) put options and (European) binary put options. How many options of what kind do you have to buy/sell?