Examination:	**Macroeconomic Analysis (No. 1428)**
Semester:	**Summer Semester 2006**
Examiner:	**Prof. Dr. Gerhard Schwödiauer**
Time available:	**120 min.**
The following aids may be used:	**None.**

Original

Examination Problems (1.–3.):

1. a) Derive the IS-curve

$$y_t - \bar{y} = -a\,(r_t - \bar{r}) + u_t$$

from $Y_t = C_t + I_t + G_t$ and explain, using a geometrical sketch, the notion of "natural interest rate" ($\bar{r}$). How is the real rate of interest r_t related to the nominal rate of interest i_t?

b) Assume that the central bank has a target rate of interest $i^* = \bar{r} + \pi^*$ and sets the current interest rate i_t in such a manner that $(i_t - i^*)$ is a linear function of $\pi_{t-1} - \pi^*$. The inflation expectation for period $t+1$, ${}_t\pi^e_{t+1}$, is formed before the private economic agents observe the price level for period t, and is "static" (i. e., equal to the most recent observation).
Derive and sketch geometrically the aggregate-demand function!

c) Assume that the output gap $y_t - \bar{y}$ is a linearly increasing function of the price-level forecast error $p_t - {}_{t-1}p^e_t$, where p_t is the natural logarithm of the price level, and the inflation expectation ${}_{t-1}\pi^e_t$ (formed after p_{t-1} has been observed) is "static".
Explain the economic rationale behind such an aggregate supply function!

d) Derive the difference equations describing the movements of real GDP and inflation rates after a long-run equilibrium in period 0 is disturbed by a positive demand shock $u_1 > 0$ in period 1 (and if there are no further shocks, $u_t = 0$ for $t = 2, 3, \ldots$). Solve the equations to check whether (and for which parameter values of the interest policy rule) the system returns to the previous equilibrium! Sketch the adjustment process by means of the usual AS-AD-graph!

e) Discuss how the price levels p_t, p_{t+1}, ... are determined in this model!

2. a) Derive the Cagan-equation

$$m_t - p_t = -a\,({}_tp^e_{t+1} - p_t) + u_t,$$

with $a > 0$ and $u_t \sim$ i.i.d. $(\bar{u}, \sigma^2_u)$, from the standard Keynesian LM-equation and explain the assumptions you make!

b) Assume the money supply process

$$m_t = \bar{m} + v_t ,$$

with $\bar{m} > 0$ and $v_t \sim$ i.i.d. $(0, \sigma^2_v)$, and that the price expectations ${}_t p^e_{t+1}$ are formed according to the adaptive expectations hypothesis.
Solve the Cagan-model for the (long-run) equilibrium price level p_t !

(Hint: Derive the stochastic first-order difference equation, describing the movement of prices, using the Koyck-transformation!)
Calculate the long-run (asymptotic) expected value and variance of p_t!

How does a permanent increase of the money supply at time t_0 affect the price level in the short run (i. e., in period t_0, t_{0+1} , etc.) and in the very long run?

c) Replace in the above model the adaptive expectations hypothesis by the rational expectations hypothesis ${}_t p^e_{t+1} = E_t p^e_{t+1}$, solve the Cagan-model and answer all the questions from (b)!
Compare the solutions obtained for rational expectations with those for adaptive expectations!

3. In an economy with constant population, the representative individual is endowed with 1 labor unit per period and maximizes the utility function

$$\sum_{t=0}^{\infty} \beta^t \ln c_t$$

subject to the budget constraint

$$s_t + c_t \leq w_t n_t + r_t k_t ,$$

where s_t is real saving, n_t labor supply, k_t the average real capital stock per individual; w_t and r_t are the real wage and, resp., real rental rate of capital. Real capital is accumulated by the individual households and rented by profit maximizing, price-taking firms which all produce with the same Cobb-Douglas production function. Labor productivity is growing at a constant rate of Harrod-neutral technical progress. Factor prices adjust so that labor and capital are always fully employed. Capital does not depreciate.

a) Derive the first-order conditions, in particular the difference equations describing the equilibrium dynamics of real consumption and capital accumulation!

b) Give the conditions for a steady-state equilibrium with positive capital stock and consumption and show geometrically (by means of a phase portrait) that such an equilibrium is unstable and a "saddle point"!

c) Sketch in the phase portrait the saddle-path trajectory and, by means of these tools, the adjustment of the economy after a positive permanent shock to total factor productivity!

– End of text. Good luck! –