| | |
|---|---|
| **Examination:** | **Macroeconomic Analysis (No. 1428)** |
| **Semester:** | **Winter Semester 2006/07** |
| **Examiner:** | **Prof. Dr. Gerhard Schwödiauer** |
| **Time available:** | **120 min.** |
| **The following aids may be used:** | **None.** |

**Examination Problems (1. and 2.):**

1. a) Derive the IS-curve

$$y_t - \bar{y} = - a\,( r_t - \bar{r} ) + u_t$$

from $Y_t = C_t + I_t + G_t$ and explain, using a geometrical sketch, the notion of "natural interest rate" ($\bar{r}$). How is the real rate of interest $r_t$ related to the nominal rate of interest $i_t$?

b) Assume that the central bank has a target rate of interest $i^* = \bar{r} + \pi^*$ and sets the current interest rate $i_t$ in such a manner that $(i_t - i^*)$ is a linear function of $\pi_{t-1} - \pi^*$. The inflation expectation for period $t+1$, ${}_t\pi^e_{t+1}$, is formed after the private economic agents observe the price level for period $t$, and is "static" (i. e., equal to the most recent observation).
Derive and sketch geometrically the aggregate-demand function!

c) Assume that the output gap $y_t - \bar{y}$ is a linearly increasing function of the price-level forecast error $p_t - {}_{t-1}p^e_t$, where $p_t$ is the natural logarithm of the price level, and the inflation expectation ${}_{t-1}\pi^e_t$ (formed after $p_{t-1}$ has been observed) is "static".
Explain the economic rationale behind such an aggregate supply function!

d) Derive the difference equations describing the movements of real GDP and inflation rates after a long-run equilibrium in period 0 is disturbed by a positive demand shock $u_1 > 0$ in period 1 (and if there are no further shocks, $u_t = 0$ for $t = 2, 3, \ldots$). Solve the equations to check whether (and for which parameter values of the interest policy rule) the system returns to the previous equilibrium! Sketch the adjustment process by means of the usual AS-AD-graph!

e) Discuss how the price levels $p_t$, $p_{t+1}$, … are determined in this model!

2. In an economy with constant population, the representative individual is endowed with 1 labor unit per period and maximizes the utility function

$$\sum_{t=0}^{\infty} \beta^{t} \ln c_t$$

subject to the budget constraint

$$s_t + c_t \leq w_t n_t + r_t k_t ,$$

where $s_t$ is real saving, $n_t$ labor supply, $k_t$ the average real capital stock per individual; $w_t$ and $r_t$ are the real wage and, resp., real rental rate of capital. Real capital is accumulated by the individual households and rented by profit maximizing, price-taking firms which all produce with the same Cobb-Douglas production function. Labor productivity is growing at a constant rate of Harrod-neutral technical progress. Factor prices adjust so that labor and capital are always fully employed. Capital does not depreciate.

a) Derive the first-order conditions, in particular the difference equations describing the equilibrium dynamics of real consumption and capital accumulation!

b) Give the conditions for a steady-state equilibrium with positive capital stock and consumption and show geometrically (by means of a phase portrait) that such an equilibrium is unstable and a "saddle point"!

c) Sketch in the phase portrait the saddle-path trajectory and, by means of these tools, the adjustment of the economy after a positive permanent shock to total factor productivity!

*– End of text. Good luck! –*