

Examination: 20117 Mathematics for Business

30.07.2011

Examiner: Prof. Dr. K. Deckelnick

Time allowed: 60 minutes

The following aids are allowed:

Five double sided sheets of paper with the main results of the course

Electronic calculator

Dictionary

Please answer three of the following four questions. If you should attempt all four questions, then the marks for the best three answers will be used.

1. (16 points) Consider the function $f(x) = \sqrt{x}(x-1)$.

(a) Write down the maximal subset of \mathbb{R} on which f can be defined.

(b) Find the local extreme points and sketch the graph of f .

(c) Calculate $\int_1^4 f(x)dx$. (Hint: Use a suitable substitution).

2. (16 points) Let $f(x, y) = e^{2x}(x + y^2 + 2y)$.

(a) Determine and classify the stationary points of f .

(b) Assume that the solutions of the equation $f(x, y) = 0$ can be written in the form $y = h(x)$ when (x, y) is close to $(0, 0)$. Find $h'(0)$.

3. (16 points) Examine the following problem with the help of Lagrange's method:

$$\text{maximize } 3x^{\frac{1}{3}}y^{\frac{4}{3}} \quad \text{subject to } 10x + 20y = 3000.$$

(a) Determine the only solution (x^*, y^*, λ^*) of the necessary first order conditions.

(b) By approximately how much will the optimal value increase if the constant in the constraint is increased from 3000 to 3000.5?

4. (16 points) Let

$$A = \begin{pmatrix} 2 & \alpha & -2 \\ -1 & -2 & -1 \\ 0 & 1 & \alpha \end{pmatrix},$$

where $\alpha \in \mathbb{R}$ is a parameter.

(a) Calculate the determinant of A . For which values of α is the matrix A invertible?

(b) Choose $\alpha = 2$ and determine the set of solutions of the linear system $Ax = b$, where

$$b = \begin{pmatrix} 2 \\ 1 \\ -2 \end{pmatrix}.$$