

Question 1: General equilibrium with production (10 points)

Consider an economy with N firms and N households. Firm 1 is owned by household 1, firm 2 by household 2, and so on. All firms produce the same consumption good, but they use different technologies. The technology of firm i is described by the production function

$$f_i(L) = \frac{1}{i}L$$

where L denotes the labour input (thus $f_1(L) = L, f_2(L) = \frac{1}{2}L, \dots, f_N(L) = \frac{1}{N}L$).

All household have the same preferences. Letting x_i denote the quantity of the consumption good consumed by household i and l_i the amount of time consumed as leisure, these preferences can be represented by the utility function

$$u(x_i, l_i) = \alpha \ln(x_i) + (1 - \alpha) \ln(l_i), 0 < \alpha < 1$$

Initially, each household is endowed with one unit of time that can either be consumed as leisure or sold as labour.

1. What can you deduce from the first theorem of welfare economics about which firms will produce a positive output quantity in equilibrium?
2. Determine a Walrasian equilibrium for this economy!

Question 2: The core of an exchange economy (8 points)

Consider an exchange economy with $k = 2$ and $n = 3$ and the following preferences:

$$\begin{aligned} u_1(x_1^1, x_1^2) &= \min\{x_1^1, x_1^2\} \\ u_2(x_2^1, x_2^2) &= x_2^1 + x_2^2 \\ u_3(x_3^1, x_3^2) &= x_3^1 + x_3^2 \end{aligned}$$

Furthermore, $\omega_1 = (1, 0)$ and $\omega_2 = \omega_3 = (0, 1)$. Determine the core of this economy!

Question 3: Decisions under uncertainty (8 points)

John's preferences for wealth can be represented by the expected utility function $u(x) = \sqrt{x}$, where x denotes his discounted life income.

If he decides to become professor of economics (he will, as you, finish his university studies with an excellent diploma), his discounted life income will equal \$810,000 with certainty. If he decides to become an investment banker, his discounted life income will be \$9,000,000 if he will succeed to become a partner in a Wall Street firm, but only \$250,000 if he will fail to become one. The success probability is 20%.

1. Which alternative is preferred by John?
2. Now John is told that Mr. Smith can determine after only a short interview whether John will become a partner or not if he will go into investment banking. Mr. Smith's judgement is unfailing, but unfortunately he demands \$320,000 for it. Should John pay this money before deciding about his profession and then take his decision on the basis of Mr. Smith's forecast? (Note: $\sqrt{8,680,000} \approx 2,946$)

Question 4: Adverse selection (8 points)

Assume that there is a continuum of managers whose ability v is uniformly distributed on the unit interval $[0,1]$. If employed by a firm, a manager with ability v will bring an expected additional profit of v minus his pay. If the manager is not employed and opens his own business, he can earn v^2 , which consequently is his reservation utility. Firms cannot determine the ability of a manager beforehand, whereas the manager knows his reservation utility. Assume that firms must offer their managers a fixed pay w independent of their profit. At the equilibrium wage, the demand for managers and the supply equal.

1. Determine an equilibrium wage $w > 0$!
2. Comment on the welfare effects of this wage!

Bonne chance!