# Exam - 14.02.2009

**Relax and take a deep breath** • You are allowed 1 hour for your work • For full grade, you must solve all questions • All questions are of equal value. Their various parts, though, are not of equal weight • In your answers, you must justify your claims • The use of a calculator is permitted.

## Part I

*(30 Points)*

(1) Find the minimax equilibrium of the following game:

| $1 \setminus 2$ | $A$ | $B$ | $C$ |
|---|---|---|---|
| $A$ | $1,-1$ | $2,-2$ | $-2,2$ |
| $B$ | $-2,2$ | $-1,1$ | $0,0$ |
| $C$ | $2,-2$ | $0,0$ | $-1,1$ |

(2) Is this game fair? Give a precise definition of a fair game!

(3) Which side-payment is necessary in order to turn this game into a fair one?

(4) Manipulate the payoffs, so that $\tilde{q} = \left(0, \frac{1}{3}, \frac{2}{3}\right)$ in equilibrium!

(5) Proof that $m_i = M_j$ in <u>any</u> two-player zero-sum game ($i, j = 1, 2$ and $i \neq j$).

## Part II

*(30 Points)*

(1) Consider the following simultaneous-move game:

| I\II | $W$ | $X$ | $Y$ | $Z$ |
|---|---|---|---|---|
| $A$ | $4,5$ | $8,7$ | $5,8$ | $2,9$ |
| $B$ | $7,6$ | $6,3$ | $2,4$ | $4,8$ |
| $C$ | $5,2$ | $9,4$ | $6,5$ | $3,1$ |
| $D$ | $9,2$ | $10,1$ | $2,2$ | $5,4$ |

(a) Show that the strategies $A, B, W$ and $X$ are (iteratively) strictly dominated, by finding, in each case, a strictly dominating strategy.

(b) Find all pure-strategy Nash equilibria of the game!

(c) Graph the four different payoffs for each player in the payoff-space! Graph also the inducement correspondence for both players in the same figure!

(c) Find the mixed-strategy Nash equilibrium of the game!

(2) Is it possible that a mixed strategy is strictly dominated by a pure strategy even though it assigns positive probability only to pure strategies that are not strictly dominated? If yes, give an example! If not, proof!

– See overleaf –

(3) Is it possible that rationalizable strategy fails to be a best response, given only pure strategies of the opponents? If yes, give an example. If not, proof!

# Part III

***(30 Points)***

(1) In a *Bertrand duopoly*, two firms, $i, j = 1, 2$, compete by choosing simultaneously and independently the prices $p_i, p_j \geq 0$ at which they will sell their output. Each firm's cost function is $C(q_i) = 5\,q_i$, that is, each firm has a constant marginal cost $c_i = 5$. Finally, the market demand is such that the firm that offers the lower price, $p = \min\{p_i, p_j\}$ captures the entire demand, $q = 100 - p$; whereas if the prices are equal, the two firms split the market.

(a) What is firm $i$'s profit as a function of $p_i$ and $p_j$? If firm $i$ were a monopolist, what would be its optimal price choice, $p_M$?

(b) What are the firms' best response functions? Graph them in the same axes.

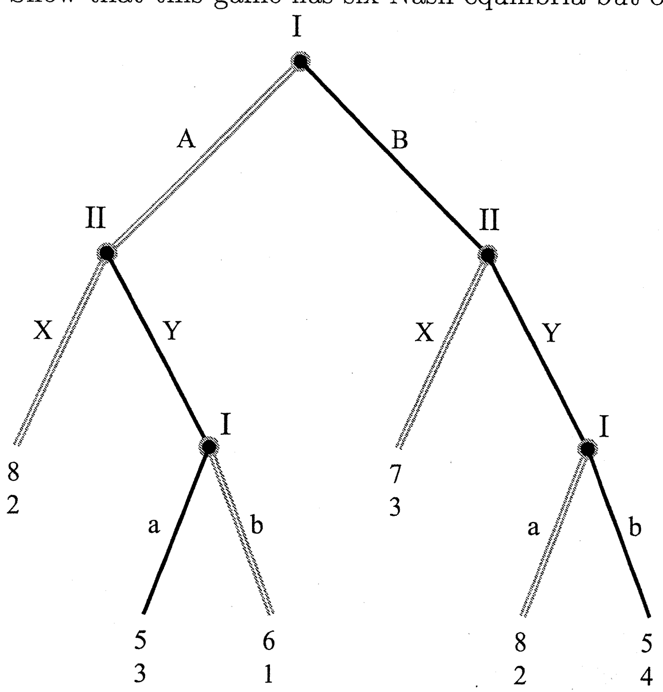
(c) Find the Nash equilibrium of the game. What is the market price in equilibrium? What are the firms' profits?

(2) Give an example for a game of strategic complements and plain substitutes. Graph the best response functions and the iso-payoff-curves for two players for this case. Do the same (example <u>and</u> graph) for a game of strategic substitutes and plain complements!

# Part IV

***(30 Points)***

(1) Show that this game has six Nash-equilibria but only one subgame-perfect equilibrium:




(2) Create a game-tree example for a non-credible threat!

(3) Create a game-tree example for a non-credible promise!