Exam - 17.07.2010

Relax and take a deep breath • You are allowed 2 hours for your work • For full grade, you must solve all questions • All questions are of equal value. Their various parts, though, are not of equal weight • In your answers, you must justify your claims • The use of a calculator is permitted.

Part I *(30 Points)*

(1) Consider the following zero-sum game:

1 \ 2	A	B	C
A	$1, -1$	$-1, 1$	$2, -2$
B	$-3, 3$	$4, -4$	$-4, 4$
C	$2, -2$	$0, 0$	$-1, 1$

(a) Find the minimax equilibrium of the following game:

(b) Is this game symmetric? Give a precise definition of a symmetric game!

(c) Is this game fair? Give a precise definition of a fair game!

(d) Which side-payment is necessary in order to turn this game into a fair one?

(e) Manipulate the payoffs, so that it becomes a symmetric game!

(2) Proof that $m_i = -M_j$ in any two-player zero-sum game ($i, j = 1, 2$ and $i \neq j$).

Part II *(30 Points)*

(1) Consider the following simultaneous-move game:

1 \ 2	W	X	Y	Z
A	$0, 7$	$2, 5$	$7, 0$	$0, 1$
B	$5, 2$	$3, 3$	$5, 2$	$0, 1$
C	$7, 0$	$2, 5$	$0, 7$	$0, 1$
D	$0, 0$	$0, -2$	$0, 0$	$1, -1$

(a) Give a precise definition of a strictly dominated strategy!

(b) Show that the strategies D and Z are (iteratively) strictly dominated! Find, in each case, a strictly dominating strategy!

(c) Give a precise definition a rationalizable strategy!

(d) Which of the strategies of the game above are rationalizable? Why?

(e) Find all pure-strategy Nash equilibria of the game!

(2) Is it possible that a mixed strategy is strictly dominated by a pure strategy even though it assigns positive probability only to pure strategies that are not strictly dominated? If yes, give an example! If not, proof!

(3) Is it possible that a rationalizable strategy fails to be a best response, given only pure strategies of the opponents? If yes, give an example. If not, proof!

Part III

(30 Points)

(1) In the *private provision of a public good*, two players (1, 2) decide whether to spend some of their private good x_i in order to supply a public G. Player i's supply of the public good is g_i, with $g_1 + g_2 = G$. Prices are normalized to 1 and player i's initial endowment is $\omega_i > 0$. Both players have preferences over their private good (x_i for player i) and G : $u_1(x_1, G) = x_1 + \alpha \log(G+1)$, $u_2(x_2, G) = x_2 + \log(G+1)$, with $\alpha > 0$.

(a) Is this a game of plain substitutes or plain complements? Is it a game of strategic substitutes or strategic complements?

(b) Find the best response function for both players!

(c) Graph the two players' best response functions for each of the following three cases: $\alpha \in (0, 1)$, $\alpha = 1$ and $\alpha > 1$!

(d) Find the pure strategy Nash-equilibrium for $\alpha = 2$!

(2) Give an example for a game of plain complements and strategic substitutes. Graph the best response functions and the iso-payoff-curves for two players for this case. Do the same (example and graph) for a game of plain substitutes and strategic complements!

Part IV

(30 Points)

(1) Specify all terminal nodes!

(2) Specify all non-terminal nodes!

(3) How many strategies does player I (II) have?

(4) Which strategies are payoff-equivalent?

(5) Specify all Nash-equilibria in pure strategies!

(6) What is the subgame-perfect equilibrium of the game?