Examination: 20023 – Option Pricing Theory **Summer Term 2008**
Examiner: Prof. Dr. Peter Reichling
Time available: 60 minutes

Aids permitted: non-programmable pocket calculators; English dictionaries without any markings. The examination is comprised of **three** problems. All of them are to be solved. Answers must be given in **English**. **Good luck!**

Examination Questions (60 Points Total):

Problem 1 (Binomial Model – 23 Points)

A stock price quotes currently at \$100 per share and can either rise by 15% or fall by 15% (per month) within the next three months. The (continuously compounded) risk-free interest rate is 3.922% p.a.

a) Using a binomial tree, show the possible stock price development. (3 points)

b) Determine the current value of an American call option with a strike price of \$110 and a maturity of three months. Demonstrate the possible option price development with the help of a binomial tree. (8 points)

c) Determine the prices of a European put and a European call options on the same stock with the strike price of \$110 and maturity of one month. Check if the put-call parity holds. (5 points)

d) Suppose the put option you have considered in c) can be purchased for \$10. Check if the put-call parity is violated. If so, with the help of an arbitrage table demonstrate how to execute a "free lunch" arbitrage strategy. (7 points)

Problem 2 (Black-Scholes Model – 27 Points)

A stock has a current price of \$80 per share and a volatility of 20%. The (continuously compounded) risk-free interest rate equals 3.6% p.a.

a) Within the framework of the Black-Scholes model determine the price of a European put as well as a European call option with an exercise price of \$75 and maturity of six months. (7 points)

b) Suppose you have short positions in 80,000 put options and 30,000 call options you have considered in a). Check if this portfolio is delta-neutral. If it is not delta-neutral, use options to construct a delta-neutral portfolio. What is the value of this portfolio? (10 points)

c) Suppose that, having the delta-neutral portfolio from b), you anticipate a large change in the stock price. Assuming that the gamma of put and call equals 0.035, check if this portfolio is still protected. If not, show what compositional adjustments should be made. (6 points)

d) Is it possible to use the standard Black-Scholes model to value American put options? Why or why not? Explain briefly. (4 points)

Problem 3 (Exotic Options - 10 Points)

a) Demonstrate formally and graphically the payoff profiles of European cash-or-nothing and asset-or-nothing put and call options. (4 points)

b) Show how European call and put options can be replicated using cash-or-nothing and asset-or nothing options. Derive the value of an asset-or-nothing put and call. (6 points)

Distribution Function for the Standard Normal Distribution for Non-Negative Arguments

x	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7034	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767