Examination: 20023 – Option Pricing Theory **Summer Term 2010**

Examiner: Prof. Dr. Peter Reichling

Time available: 60 minutes

Aids permitted: non-programmable pocket calculators;
English dictionaries without any markings.

The examination comprises **two** problems. All of them are to be solved. Answers must be given in **English**. Good luck!

Examination Questions (60 Points Total):

Problem 1 (Binomial Model – 25 Points)

A stock price quotes currently at \$50 per share and can either rise or fall by 12% (per period) within the next two months. The (discretely compounded) risk-free interest rate is 3% p.a.

a) Using a binomial tree, show the possible stock price development. (2 points)

b) Determine the current value of a European put and a European call option with strike price \$45 and maturity two months. (8 points)

c) Suppose that a European **put** option with strike price \$45 and maturity two months can be purchased at the market for \$3. With the help of an arbitrage table, demonstrate how to execute an arbitrage strategy in order to receive a profit today. (7 points)

d) Show how to derive the binomial pricing formula for a European **call** option (8 points)

Problem 2 (Black-Scholes Model – 35 Points)

Stock X shows a current price of \$28 per share and a volatility of 25%. The (continuously compounded) risk-free interest rate equals 2% p.a.

a) Within the framework of the Black-Scholes model determine the price, the delta, and the gamma of a European put and a European call option on the above-mentioned stock with exercise price \$25 and maturity 65 days (assume a 365-days year). Interpret the values you received for delta and gamma. (13 points)

b) Compute the omega of the put and the call option in a) and interpret this measure. (6 points)

c) Suppose that you have written 20 **call** options on stock X with exercise price \$25 and maturity 65 days. To hedge your option position construct a self-financing portfolio which is insensitive to small changes in the price of the underlying stock. What position (short/long) and how many stocks will be required? How much money will you borrow/lend? (4 points)

d) Suppose that, having written 20 call options, you want to construct a self-financing portfolio which is insensitive not only to small but also to larger changes in the price of the underlying stock. What position (short/long) and how many options and stocks will be required? How much money will you borrow/lend? (7 points)

e) Explain in details the principle of risk-neutral valuation. (5 points)

Distribution Function of the Standard Normal Distribution for Non-negative Arguments

x	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7034	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767