Examiner: Prof. Dr. Peter Reichling
Time available: 60 minutes
Aids permitted: non-programmable pocket calculators;
English dictionaries without any markings.
The examination comprises three problems. All of them are to be solved. All numbers are to be rounded to two decimal places. Answers to all problems must be given in English. Good luck!

## Examination Questions ( 60 Points Total):

## Problem 1 (Binomial Model / Put-Call Parity - 25 Points)

Consider two stocks, A and B, both trading at a price of \$20. Furthermore, consider a oneperiod binomial model (i.e. a period is a month) in which stock A's price can take one of the following values after one period $\{35 ; 5\}$. Stock B’s price can take one of the following values after one period $\{36 ; 18\}$. The continuously compounded risk-free rate is $10 \%$ p.a.
a) Using replication, find the prices of 1-month at-the-money call options on stocks A and B. (7 points)
b) Which call option, that on stock A or that on stock B, is worth more? Why? (4 points)
c) Find the value of a long straddle position on stock A. (4 points)
d) Now consider a two-period binomial model where each period is one month. Find the value of a chooser option on stock A where at the choice date $t=1$ the holder of the option must choose whether the option is a call or a put. Use a strike price for the call and the put of 20. (10 points)

## Problem 2 (Black-Scholes Model - 20 Points)

Consider the following Black-Scholes parameters: $\mathrm{S}=100, \mathrm{~K}=100, \mathrm{~T}=1, \sigma=30 \%, \mathrm{r}=10 \%$ and $G=90$.
a) Write down the payoff profile of a gap call option. What role does K play in a gap call option? (3 points)
b) Show how to synthetically create a gap call option with other options. What option types will you use and how many options of each type will you need? (7 points)
c) Use the Black-Scholes model to find the price of the gap call option. (6 points)
d) Consider the case where $<S_{T}<G$. What would be the implication for the gap call option's payoff? What would be the implication for the option's premium? (3 points)

## Problem 3 (Greeks - 15 Points)

Consider a stock currently trading at 100. An at-the-money call option with maturity of three months has the following price and Greeks: $\mathrm{C}=5.598, \Delta=0.565$ and $\Gamma=0.032$.
a) If the stock price jumps to 120 , what is the new call value predicted by the delta alone? What is the new call value predicted by the delta and gamma combined? Which approximation is likely to be more accurate? Why? (9 points)
b) Assume that you have a short position in 100 call options. What would you do to make the position delta-neutral? (2 points)
c) How would your answer from b) change if you had a short position in 100 put options instead? (4 points)

Distribution Function for the Standard Normal Distribution for Non-Negative Arguments

| $\mathbf{x}$ | $\mathbf{0 . 0 0}$ | $\mathbf{0 . 0 1}$ | $\mathbf{0 . 0 2}$ | $\mathbf{0 . 0 3}$ | $\mathbf{0 . 0 4}$ | $\mathbf{0 . 0 5}$ | $\mathbf{0 . 0 6}$ | $\mathbf{0 . 0 7}$ | $\mathbf{0 . 0 8}$ | $\mathbf{0 . 0 9}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathbf{0 . 0}$ | 0.5000 | 0.5040 | 0.5080 | 0.5120 | 0.5160 | 0.5199 | 0.5239 | 0.5279 | 0.5319 | 0.5359 |
| $\mathbf{0 . 1}$ | 0.5398 | 0.5438 | 0.5478 | 0.5517 | 0.5557 | 0.5596 | 0.5636 | 0.5675 | 0.5714 | 0.5753 |
| $\mathbf{0 . 2}$ | 0.5793 | 0.5832 | 0.5871 | 0.5910 | 0.5948 | 0.5987 | 0.6026 | 0.6064 | 0.6103 | 0.6141 |
| $\mathbf{0 . 3}$ | 0.6179 | 0.6217 | 0.6255 | 0.6293 | 0.6331 | 0.6368 | 0.6406 | 0.6443 | 0.6480 | 0.6517 |
| $\mathbf{0 . 4}$ | 0.6554 | 0.6591 | 0.6628 | 0.6664 | 0.6700 | 0.6736 | 0.6772 | 0.6808 | 0.6844 | 0.6879 |
|  |  |  |  |  |  |  |  |  |  |  |
| $\mathbf{0 . 5}$ | 0.6915 | 0.6950 | 0.6985 | 0.7019 | 0.7034 | 0.7088 | 0.7123 | 0.7157 | 0.7190 | 0.7224 |
| $\mathbf{0 . 6}$ | 0.7257 | 0.7291 | 0.7324 | 0.7357 | 0.7389 | 0.7422 | 0.7454 | 0.7486 | 0.7517 | 0.7549 |
| $\mathbf{0 . 7}$ | 0.7580 | 0.7611 | 0.7642 | 0.7673 | 0.7704 | 0.7734 | 0.7764 | 0.7794 | 0.7823 | 0.7852 |
| $\mathbf{0 . 8}$ | 0.7881 | 0.7910 | 0.7939 | 0.7967 | 0.7995 | 0.8023 | 0.8051 | 0.8078 | 0.8106 | 0.8133 |
| $\mathbf{0 . 9}$ | 0.8159 | 0.8186 | 0.8212 | 0.8238 | 0.8264 | 0.8289 | 0.8315 | 0.8340 | 0.8365 | 0.8389 |
|  |  |  |  |  |  |  |  |  |  |  |
| $\mathbf{1 . 0}$ | 0.8413 | 0.8438 | 0.8461 | 0.8485 | 0.8508 | 0.8531 | 0.8554 | 0.8577 | 0.8599 | 0.8621 |
| $\mathbf{1 . 1}$ | 0.8643 | 0.8665 | 0.8686 | 0.8708 | 0.8729 | 0.8749 | 0.8770 | 0.8790 | 0.8810 | 0.8830 |
| $\mathbf{1 . 2}$ | 0.8849 | 0.8869 | 0.8888 | 0.8907 | 0.8925 | 0.8944 | 0.8962 | 0.8980 | 0.8997 | 0.9015 |
| $\mathbf{1 . 3}$ | 0.9032 | 0.9049 | 0.9066 | 0.9082 | 0.9099 | 0.9115 | 0.9131 | 0.9147 | 0.9162 | 0.9177 |
| $\mathbf{1 . 4}$ | 0.9192 | 0.9207 | 0.9222 | 0.9236 | 0.9251 | 0.9265 | 0.9279 | 0.9292 | 0.9306 | 0.9319 |
|  |  |  |  |  |  |  |  |  |  |  |
| $\mathbf{1 . 5}$ | 0.9332 | 0.9345 | 0.9357 | 0.9370 | 0.9382 | 0.9394 | 0.9406 | 0.9418 | 0.9429 | 0.9441 |
| $\mathbf{1 . 6}$ | 0.9452 | 0.9463 | 0.9474 | 0.9484 | 0.9495 | 0.9505 | 0.9515 | 0.9525 | 0.9535 | 0.9545 |
| $\mathbf{1 . 7}$ | 0.9554 | 0.9564 | 0.9573 | 0.9582 | 0.9591 | 0.9599 | 0.9608 | 0.9616 | 0.9625 | 0.9633 |
| $\mathbf{1 . 8}$ | 0.9641 | 0.9649 | 0.9656 | 0.9664 | 0.9671 | 0.9678 | 0.9686 | 0.9693 | 0.9699 | 0.9706 |
| $\mathbf{1 . 9}$ | 0.9713 | 0.9719 | 0.9726 | 0.9732 | 0.9738 | 0.9744 | 0.9750 | 0.9756 | 0.9761 | 0.9767 |

