Examiner: Prof. Dr. Peter Reichling

You are welcome to use non-programmable pocket calculators as well as English language dictionaries without any markings. This examination comprises 3 problems (on 3 pages). All of the problems are to be solved. Derivations of the formulas from the lecture or the exercise are not (!) required.
Good luck!

Examination Questions (Total Number of Points: 60)

Problem 1 (Value at Risk of a Bond Portfolio – 21 Points)

Consider a bond portfolio with the following positions:

Long positions in two coupon bonds, CB1 and CB2, where CB1 has a face value of 150,000€, coupon of 10 % p.a., CB2 has a face value of 220,000€, coupon of 4 % p.a. Both coupon bonds have a maturity of two years and make annual coupon payments. Furthermore, there is a short position in two zero bonds, ZB1 and ZB2, where ZB1 has a face value of 170,000€ and a maturity of one year, ZB2 has a face value of 100,000€ and a maturity of two years.

(a) Explain why the spot rates are appropriate risk factors in assessing the risk of the bond portfolio. (1 point)

Assume that the current 1-year spot rate equals 4.7 % p.a. and the current 2-year spot rate equals 5.2 % p.a.

(b) Compute the current value of the bond portfolio. (3 points)

(c) Develop a linear approximation between small changes in the spot rates and the appropriate changes in the value of the bond portfolio. (4 points)

Assume that daily changes in the two given spot rates are bivariate normally distributed with a mean of zero, a standard deviation of 0.072 % of the daily change in the 1-year spot rate, a standard deviation of 0.055 % of the daily change in the 2-year spot rate, and a correlation coefficient of 0.61 between the daily changes of the two spot rates.

(d) Compute overnight values at risk of the bond portfolio for confidence levels of 95 % and 99 % by using your linear approximation from part (c). (7 points)

Assume that daily changes in spot rates are identically and independently distributed.

(e) Compute the corresponding values at risk for an assumed holding period of ten trading days. (2 points)

(f) Write down the assumptions of the value at risk computation by reconsidering (c), (d), and (e). (4 points)

— See over leaf —

Problem 2 (Variance-Covariance Approach vs. Diagonal Model – 19 Points)

Consider a stock portfolio consisting of 6,000 shares of company A (8€ per share), 2,000 shares of company B (14€ per share), and 1,500 shares of company C (18€ per share). All three stocks belong to the ABC All Share market index (M). The following statistical information of the stocks and the index is known: the volatility of A is 75%, the volatility of B is 50%, the volatility of C is 110%, and the volatility of ABC is 55%. The matrix of correlation coefficients between the rates of return of two stocks is given below, where, e.g., $\rho_{(M,A)}$ denotes the correlation coefficient between the rates of return of the ABC market index and stock A.

$$\rho = \begin{pmatrix} 1 & 0.6 & 0.75 & 0.88 \\ \rho_{(B,A)} & 1 & 0.2 & 0.77 \\ \rho_{(C,A)} & \rho_{(C,B)} & 1 & 0.425 \\ \rho_{(M,A)} & \rho_{(M,B)} & \rho_{(M,C)} & 1 \end{pmatrix}$$

Compute the value at risk (in €) of the portfolio for an assumed holding period of two weeks and a confidence level of 97.5% using

(a) the variance-covariance approach

(b) the diagonal model

and interpret the results.

Problem 3 (Discriminative Power of Rating Functions – 20 Points)

At time $t = 0$ rating agencies "Coincidence" and "Fortuity" carried out ratings of 100 companies, both using 4 rating classes but different rating functions. Observations at time $t = 1$ lead to the following contingency table of agency "Coincidence":

	Observation at time $t = 1$	
Rating at time $t = 0$	Default	Non-default
D	9	21
C	7	23
B	3	17
A	1	19

In case of agency "Fortuity", the following contingency table results:

	Observation at time $t = 1$	
Rating at time $t = 0$	Default	Non-default
d	8	16
c	6	22
b	2	12
a	4	30

(a) Use one (and the same) diagram to show the cumulative accuracy profile (CAP) curve of

(i) the rating function of agency "Coincidence",

(ii) the rating function of agency "Fortuity",

(iii) the perfect rating function,

(iv) the random rating function.

Compute (or just write down) all 4 (!) accuracy ratio (AR) values. (16 points)

(b) Do the rating functions of agencies "Conicidence" and "Fortuity" exhibit discriminative power? Which rating function can differentiate better between companies of high and low creditworthiness? Justify your answers. (4 points)

Distribution Function N(x) of the Standard Normal Distribution for Non-negative Arguments x

x	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7034	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9773	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9983	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9990	0.9993	0.9995	0.9997	0.9998	0.9998	0.9999	0.9999	1.0000