

Examiner: Prof. Dr. Peter Reichling

You are welcome to use non-programmable pocket calculators as well as English language dictionaries without any markings. This examination comprises 4 problems (on 3 pages). All of the problems are to be solved. Derivations of formulas from the lecture or the exercise are not (!) required, but your answers have to be reasonably justified.

Good luck!

Examination Questions (Total Number of Points: 60)

Problem 1 (Downside Risk – 12 points)

Consider two stocks, A and B , with normally distributed and perfectly negatively correlated returns. Stock A shows an expected return of 15% p.a. and a volatility of 15%, while stock B has an expected return of 25% p.a. and a volatility of 35%. Short selling of stocks is not allowed.

- (a) Plot possible portfolios of stocks A and B in a (μ, σ) -diagram. (2 points)
- (b) If possible, determine the shares x_A and x_B of stocks A and B , respectively, in an optimal portfolio according to ROY's criterion using a target return of 17% p.a. In case of existence, determine the shortfall probability of this portfolio, otherwise justify why such an optimal portfolio cannot exist. (2 points)
- (c) If possible, determine the shares x_A and x_B of stocks A and B , respectively, in an optimal portfolio according to KATAOKA's criterion using a shortfall probability of 44.04%. In case of existence, determine the corresponding target return of this portfolio, otherwise justify why such an optimal portfolio cannot exist. (2 points)
- (d) If possible, determine the shares x_A and x_B of stocks A and B , respectively, in an optimal portfolio according to TELSER's criterion using a target return of 10% p.a. and a shortfall probability of 36.32%. In case of existence, determine the expected rate of return and the volatility of this portfolio, otherwise justify why such an optimal portfolio cannot exist. (3 points)
- (e) If possible, determine the shares x_A and x_B of stocks A and B , respectively, in an optimal portfolio according to TELSER's criterion using a target return of 10% p.a. and a shortfall probability of 33%. In case of existence, determine the expected rate of return and the volatility of this portfolio, otherwise justify why such an optimal portfolio cannot exist. (3 points)

Problem 2 (Value at Risk of a Bond Portfolio – 12 Points)

Consider a bond portfolio where you have invested 30% of your total amount in a perpetual (consol bond), paying 50,000 € per year and furthermore you invested the remaining amount in an amortizable loan with a maturity of three years, a face value of 150,000 €, an amortization payment of 50,000 € per year and a loan-specific interest rate of 5% p.a. At the moment, a flat term structure with an interest rate level of 3% is prevalent.

- (a) Compute the current price of the bond portfolio. (3 points)
- (b) Compute the modified duration of the bond portfolio. (7 points)
- (c) Suppose we want to sell the bond portfolio in 15 days and observe an interest rate volatility of 8% at the market. How much is the value at risk of our bond portfolio for a confidence level of 97.5% when there are 250 trading days per year. (2 points)

Problem 3 (Discriminative Power of Rating Functions – 21 Points)

At time $t = 0$, rating agencies "Luck" and "Chance" carried out ratings of 70 companies, both using 3 rating classes but different rating functions. Observations at time $t = 1$ lead to the following contingency table of agency "Luck":

Rating at time $t = 0$	Observation at time $t = 1$	
	Default	Non-default
C	10	20
B	6	14
A	2	18

In case of agency "Chance", the following contingency table results:

Rating at time $t = 0$	Observation at time $t = 1$	
	Default	Non-default
c	12	18
b	6	19
a	2	13

- (a) Use one (and the same) diagram to show the receiver operating characteristic (ROC) curve of
 - (i) the rating function of agency "Luck",
 - (ii) the rating function of agency "Chance",
 - (iii) the perfect rating function,
 - (iv) and the random rating function
 and compute all 4 (!) area under curve (AUC) values. (15 points)
- (b) Compute the corresponding accuracy ratio (AR) values for "Chance" and "Luck". (2 points)
- (c) Do the rating functions of agencies "Luck" and "Chance" exhibit discriminative power? Which rating function can differentiate better between companies of high and low creditworthiness? Justify your answers. (4 points)

Problem 4 (General Questions – 15 Points)

The following multiple choice part comprises five questions. For each question, three answers are given, but only one answer is correct. You are allowed to (clearly) indicate your answers on this sheet. Every correct answer yields 3 points, every incorrect answer yields -1 points. If you do not answer a question, you will get 0 points. However, the total points for this problem cannot be negative.

- (a) The three pillars of the Basel II framework are:
- (i) Minimum capital requirements, Supervisory review, Market discipline.
 - (ii) Maximum capital requirements, Risk controlling, Supervisory review.
 - (iii) Expected capital requirements, Market discipline, Risk measurement approaches.
- (b) The model underlying the Internal Ratings-Based Approach for credit risk under the Basel II framework is:
- (i) Reduced-form model.
 - (ii) One-factor Gaussian copula model.
 - (iii) Quadratic value at risk model.
- (c) Under what assumption does second-order stochastic dominance imply first-order stochastic dominance?
- (i) If the investor is risk averse and prudent.
 - (ii) If there are no crossing points of the distribution functions.
 - (iii) Second-order stochastic dominance always implies first-order stochastic dominance.
- (d) The following two lotteries are possible: Lottery 1 has a 20% probability of winning 40, a 60% probability of winning 100 and a 20% probability of winning 160. Lottery 2 has a 10% probability of winning 60 and a 90% probability of winning 110. Is there any second-order stochastic dominance and what kind of?
- (i) Lottery 1 dominates Lottery 2 stochastically of second order.
 - (ii) Lottery 2 dominates Lottery 1 stochastically of second order.
 - (iii) There is no kind of second-order stochastic dominance.
- (e) Suppose we have a subordinated loan with a zero bond structure and a (positive) face value of $K_2 - K_1$. According to the MERTON model, which of the following duplication portfolios does **not** describe the payoff structure of the above described loan?
- (i) Long stock position, a short zero bond position with face value K_1 , a long put with strike K_1 , and a short call with strike K_2 .
 - (ii) Short stock position, a long zero bond position with face value K_2 , a short put with strike K_1 , two long calls with strike K_1 , and a short call with strike K_2 .
 - (iii) Short put with strike K_2 , a long put with strike K_1 and a long zero bond with face value $K_2 - K_1$.

**Distribution Function $N(x)$ of the Standard Normal Distribution
for Non-negative Arguments x**

x	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7034	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9773	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9983	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9990	0.9993	0.9995	0.9997	0.9998	0.9998	0.9999	0.9999	1.0000