

Examiner: Prof. Dr. Peter Reichling

You are welcome to use non-programmable pocket calculators as well as English language dictionaries without any markings. This examination comprises 3 problems (on 3 pages). All of the problems are to be solved. Derivations of formulas from the lecture or the exercise are not (!) required, but your answers have to be reasonably justified.

Good luck!

### Examination Questions (Total Number of Points: 60)

#### Problem 1 (Lower Partial Moments – 28 points)

Consider a stock whose random return  $R$  is defined by the following density function:

$$f_R(r) = \begin{cases} 80 \cdot r + 8 & , -0.1 \leq r < 0 \\ -80 \cdot r + 8 & , 0 \leq r < 0.05 \\ 4 & , 0.05 \leq r < 0.1 \\ -80 \cdot r + 12 & , 0.1 \leq r < 0.15 \\ 0 & , \text{otherwise} \end{cases}$$

- Compute the expected stock return and the standard deviation. (14 points)
- Compute the lower partial moments of order zero and one, using the expected return as the target return. Interpret the two computed lower partial moments. (10 points)
- Plot the density function of the stock return, its expected value, and the lower partial moment of order zero from part (b) in the same diagram. (4 points)

#### Problem 2 (Value at Risk of Binary Options – 18 Points)

There is a long position of 10,000 out-of-the-money cash-or-nothing European put options on the S&P500 index with a strike price of 1,100 points and a maturity of 6 months in a bank's trading book. The volatility of the index is assumed to be 18.00%. The (continuously compounded) current 6-month Euribor equals 1.225% p.a. and the current index value is 1,283 points. One cash-or-nothing put option pays 1 (currency here: index points!) if the index' closing price at maturity is below the strike price, otherwise there is no payment. The price of one cash-or-nothing put,  $P^{CoN}$ , can be calculated according to the BLACK-SCHOLES-MERTON model by the formula

$$P^{CoN} = e^{-rT} \cdot N(-d_2),$$

where  $r$  denotes the interest rate,  $T$  the time to maturity and  $N(\cdot)$  the value of the standard normal distribution for the argument in brackets.

- (a) Compute (in index points and using the table on the last page) for one option
- (i) the option's delta (5 points),
  - (ii) the option's gamma (6 points),
  - (iii) the option's volatility using the following assumptions: the S&P500 value is the only relevant risk factor; there is a quadratic relation between the change in the S&P500 value and the change in the value of the option; the change in the value of the S&P500 is normally distributed with a mean of zero. (5 points)
- (b) Use your results of part (a)(iii) and the valuation formula to compute the value at risk of the bank's total position in € (by assuming that one contract value is 5€ per index point and there are 250 trading days per year) for an assumed period of 5 trading days and a confidence level of 97.5 %. (2 points)

### Problem 3 (Loan Valuation – 14 points)

The assets of a company are worth 3.5 m. €. The volatility of total assets is 20 %. Four years ago a loan was raised at bank *A*. This loan was issued as a zero bond with a maturity of 6 years and a repayment amount of 2 m. €. There is no further debt. No additional equity or debt nor dividend payments, share buy-backs, interest payments etc. are planned. Bank *A* would like to transfer the loan to bank *B*. The current term structure of the discretely compounded (!) interest rates is:

| maturity               | 1 year | 2 years | 3 years | 4 years | 5 years | 6 years |
|------------------------|--------|---------|---------|---------|---------|---------|
| spot rate (in % p. a.) | 0.79   | 1.19    | 1.54    | 1.91    | 2.23    | 2.48    |

- (a) How can the loan of bank *A* and the equity of the company be described and illustrated in a payoff-diagram as a portfolio of (riskless) bonds and options? (7 points)
- (b) What is the value of the loan today if both banks value the loan according to MERTON (1974)? How much is the demanded rate of return of bank *B*? Compare this rate of return to the appropriate spot rate. (7 points)

Distribution Function  $N(x)$  of the Standard Normal Distribution  
for Non-negative Arguments  $x$

| $x$ | 0.00   | 0.01   | 0.02   | 0.03   | 0.04   | 0.05   | 0.06   | 0.07   | 0.08   | 0.09   |
|-----|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|
| 0.0 | 0.5000 | 0.5040 | 0.5080 | 0.5120 | 0.5160 | 0.5199 | 0.5239 | 0.5279 | 0.5319 | 0.5359 |
| 0.1 | 0.5398 | 0.5438 | 0.5478 | 0.5517 | 0.5557 | 0.5596 | 0.5636 | 0.5675 | 0.5714 | 0.5753 |
| 0.2 | 0.5793 | 0.5832 | 0.5871 | 0.5910 | 0.5948 | 0.5987 | 0.6026 | 0.6064 | 0.6103 | 0.6141 |
| 0.3 | 0.6179 | 0.6217 | 0.6255 | 0.6293 | 0.6331 | 0.6368 | 0.6406 | 0.6443 | 0.6480 | 0.6517 |
| 0.4 | 0.6554 | 0.6591 | 0.6628 | 0.6664 | 0.6700 | 0.6736 | 0.6772 | 0.6808 | 0.6844 | 0.6879 |
| 0.5 | 0.6915 | 0.6950 | 0.6985 | 0.7019 | 0.7034 | 0.7088 | 0.7123 | 0.7157 | 0.7190 | 0.7224 |
| 0.6 | 0.7257 | 0.7291 | 0.7324 | 0.7357 | 0.7389 | 0.7422 | 0.7454 | 0.7486 | 0.7517 | 0.7549 |
| 0.7 | 0.7580 | 0.7611 | 0.7642 | 0.7673 | 0.7704 | 0.7734 | 0.7764 | 0.7794 | 0.7823 | 0.7852 |
| 0.8 | 0.7881 | 0.7910 | 0.7939 | 0.7967 | 0.7995 | 0.8023 | 0.8051 | 0.8078 | 0.8106 | 0.8133 |
| 0.9 | 0.8159 | 0.8186 | 0.8212 | 0.8238 | 0.8264 | 0.8289 | 0.8315 | 0.8340 | 0.8365 | 0.8389 |
| 1.0 | 0.8413 | 0.8438 | 0.8461 | 0.8485 | 0.8508 | 0.8531 | 0.8554 | 0.8577 | 0.8599 | 0.8621 |
| 1.1 | 0.8643 | 0.8665 | 0.8686 | 0.8708 | 0.8729 | 0.8749 | 0.8770 | 0.8790 | 0.8810 | 0.8830 |
| 1.2 | 0.8849 | 0.8869 | 0.8888 | 0.8907 | 0.8925 | 0.8944 | 0.8962 | 0.8980 | 0.8997 | 0.9015 |
| 1.3 | 0.9032 | 0.9049 | 0.9066 | 0.9082 | 0.9099 | 0.9115 | 0.9131 | 0.9147 | 0.9162 | 0.9177 |
| 1.4 | 0.9192 | 0.9207 | 0.9222 | 0.9236 | 0.9251 | 0.9265 | 0.9279 | 0.9292 | 0.9306 | 0.9319 |
| 1.5 | 0.9332 | 0.9345 | 0.9357 | 0.9370 | 0.9382 | 0.9394 | 0.9406 | 0.9418 | 0.9429 | 0.9441 |
| 1.6 | 0.9452 | 0.9463 | 0.9474 | 0.9484 | 0.9495 | 0.9505 | 0.9515 | 0.9525 | 0.9535 | 0.9545 |
| 1.7 | 0.9554 | 0.9564 | 0.9573 | 0.9582 | 0.9591 | 0.9599 | 0.9608 | 0.9616 | 0.9625 | 0.9633 |
| 1.8 | 0.9641 | 0.9649 | 0.9656 | 0.9664 | 0.9671 | 0.9678 | 0.9686 | 0.9693 | 0.9699 | 0.9706 |
| 1.9 | 0.9713 | 0.9719 | 0.9726 | 0.9732 | 0.9738 | 0.9744 | 0.9750 | 0.9756 | 0.9761 | 0.9767 |
| 2.0 | 0.9773 | 0.9778 | 0.9783 | 0.9788 | 0.9793 | 0.9798 | 0.9803 | 0.9808 | 0.9812 | 0.9817 |
| 2.1 | 0.9821 | 0.9826 | 0.9830 | 0.9834 | 0.9838 | 0.9842 | 0.9846 | 0.9850 | 0.9854 | 0.9857 |
| 2.2 | 0.9861 | 0.9864 | 0.9868 | 0.9871 | 0.9875 | 0.9878 | 0.9881 | 0.9884 | 0.9887 | 0.9890 |
| 2.3 | 0.9893 | 0.9896 | 0.9898 | 0.9901 | 0.9904 | 0.9906 | 0.9909 | 0.9911 | 0.9913 | 0.9916 |
| 2.4 | 0.9918 | 0.9920 | 0.9922 | 0.9925 | 0.9927 | 0.9929 | 0.9931 | 0.9932 | 0.9934 | 0.9936 |
| 2.5 | 0.9938 | 0.9940 | 0.9941 | 0.9943 | 0.9945 | 0.9946 | 0.9948 | 0.9949 | 0.9951 | 0.9952 |
| 2.6 | 0.9953 | 0.9955 | 0.9956 | 0.9957 | 0.9959 | 0.9960 | 0.9961 | 0.9962 | 0.9963 | 0.9964 |
| 2.7 | 0.9965 | 0.9966 | 0.9967 | 0.9968 | 0.9969 | 0.9970 | 0.9971 | 0.9972 | 0.9973 | 0.9974 |
| 2.8 | 0.9974 | 0.9975 | 0.9976 | 0.9977 | 0.9977 | 0.9978 | 0.9979 | 0.9979 | 0.9980 | 0.9981 |
| 2.9 | 0.9981 | 0.9982 | 0.9983 | 0.9983 | 0.9984 | 0.9984 | 0.9985 | 0.9985 | 0.9986 | 0.9986 |
| 3.0 | 0.9987 | 0.9990 | 0.9993 | 0.9995 | 0.9997 | 0.9998 | 0.9998 | 0.9999 | 0.9999 | 1.0000 |

