

Examiner: **Prof. Dr. Peter Reichling**

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This examination consists of **6 problems** (3 pages). All of them are to be solved. You are welcome to use **non-programmable pocket calculators**. If not required explicitly, derivations of formulas are not necessary. But you have to give an explanation of all the symbols that you use. Answers can be given either in **English** or in **German**. Note: Students participating in the **English program** have to answer in **English**.

**Good luck!**Examination questions (Total number of points: 120)**Problem 1 (35 points)**

Note: The face value of all bonds is 100.

(a) Calculate the term structure from the following zero bonds:

Bond	Maturity (in years)	Price
ZB1	1	96.15
ZB2	2	89.00
ZB3	3	81.63

(b) What kind of term structure is existent?

(c) Calculate the corresponding forward rates.

(d) Calculate the term structure from the following coupon bonds with annual coupon payments:

Bond	Maturity (in years)	Coupon rate p.a.	Price
CB1	1	7.5 %	103.37
CB2	2	8.5 %	104.74
CB3	3	9 %	105.64

(e) Define the term yield to maturity. Calculate it for coupon bonds CB1 and CB2.

(f) Find the fair price (based on the term structure) of a 2-year coupon bond CB2\* with a coupon of 4 %. Calculate the yield to maturity of that coupon bond and compare it to the yield of maturity of CB2. How is the observed effect called?

(g) Calculate the yield to maturity for another 2-year coupon bond CB2\*\* with a coupon of 6 % and a price of 100.15.

(h) Show that there is an arbitrage possibility using CB2 and CB2\*\* in spite of identical maturity and yield to maturity. Which bond an investor would buy or sell?

(i) Find the fair price of CB2\*\* under the assumption, that the price of CB2 is fair.

**Problem 2 (20 points)**

(a) Assume a flat term structure at a level of 5 %. What is the duration of a coupon bond with face value 100, maturity 5 years, and an annual coupon of 5.67 %?

(b) By using your answer of (a), determine (approximately) the price of a bond if the term structure shifts upwards by 200 bps.

- (c) What would be the precise value of the bond at the duration point in time without interest rate change and after the interest rate change?
- (d) Outline the value of the bond depending on the interest rate at the duration point in time in a diagram.
- (e) Name three interpretations of the duration.
- (f) What is the duration of a floating rate note?

**Problem 3 (10 points)**

Two stocks A and B with the following data are given:

	A	B
Expected rate of return	13 %	15 %
Volatility	20 %	25 %

- (a) What must hold for the correlation coefficient  $\rho_{A,B}$  to reach a risk-free minimum variance portfolio?
- (b) Which restriction must hold for the correlation coefficient  $\rho_{A,B}$  to assure that the minimum variance portfolio does not violate a short-selling restriction?
- (c) Calculate the fraction of the stocks A and B in the minimum variance portfolio, the expected rate of return, and the volatility of the minimum variance portfolio for  $\rho_{A,B} = 0.5$ .

**Problem 4 (15 Points)**

Assume there is a market portfolio with an expected rate of return of 15 % and a volatility of 20 %. The risk-free interest rate is 10 %.

- (a) Express the equation of the SML and illustrate it graphically.
- (b) Calculate expected rate of return and beta of a mutual fund F, which consists of the market portfolio and the risk-free investment one half each.
- (c) Consider a stock S with the following data: expected rate of return 11 %, volatility 15 %, correlation with the market portfolio  $\rho_{S,M} = 1/3$ . Is S over-, under- or fairly priced?

**Problem 5 (25 Points)**

The following data are given:

	M	A	B
Expected rate of return	10 %	10 %	14 %
Volatility	20 %	25 %	30 %
Beta		0.5	1.5
Risk-free interest rate	6 %		

- (a) Calculate Jensen's alpha of the mutual funds A und B and evaluate the performance.
- (b) Calculate the Treynor index and the Sharpe index of A und B and evaluate the performance.
- (c) Show that with a positive beta a superior performance regarding the Treynor index implies a superior performance regarding Jensen's alpha. Show with a counter-example that this does not hold with regard to rankings.
- (d) Show that on completely diversified portfolios a superior performance regarding the Sharpe index implies a superior performance regarding the Treynor index. Show that this is also valid with regard to rankings.

### Problem 6 (15 Points)

The section will be evaluated in the following way: If a question is not answered, it yields zero points; if it is answered correctly, it yields one point; if it is answered incorrectly, it yields minus one point. However, if the total number of points for the section is negative, the number will be normalized to zero.

- (1) If a risk-free asset is available in a CAPM market equilibrium, all investors who buy stocks buy different portfolios of stocks.  correct  false
- (2) If no risk-free asset is available in a CAPM market equilibrium, the two fund separation is still valid.  correct  false
- (3) Two zero bonds with the same maturity need not to have the same yield on an arbitrage-free capital market.  correct  false
- (4) The higher the market interest rate the lower the duration of a zero bond.  correct  false
- (5) The yield to maturity of an 8 %-coupon bond with a (remaining) maturity of 2 years and a current price of 96.53 Euro is 9 %.  correct  false
- (6) The forward rate  $f_{1,2}$  is about 9.6 % for given spot rates  $s_1 = 4 %$  and  $s_2 = 5 %$ .  correct  false
- (7) If the term structure is flat, the yield of a coupon bond is equal to its coupon rate.  correct  false
- (8) Efficient portfolios only contain diversifiable risks.  correct  false
- (9) Investments placed below the securities market line in a  $(\mu, \beta)$ -space are under priced compared to the prices in capital market equilibrium.  correct  false
- (10) With a risk-free interest rate of 2 % and an expected rate of return of the market of 12 %, a portfolio with a beta of 0.8 has an expected rate of return of 10 % in the market equilibrium.  correct  false
- (11) Due to the diversification effect the volatility of a stock portfolio with uncorrelated returns of the single stocks is lower than the weighted average of the single volatilities.  correct  false
- (12) Arbitrary divisibility of the investment opportunities is no assumption of the perfect capital market.  correct  false
- (13) Net selectivity increases with unsystematic risk.  correct  false
- (14) The appraisal ratio adjusts Jensen's alpha by non-diversifiable risk.  correct  false
- (15) In the case that an investor considers the systematic risk of an investment, a positive Treynor index means a superior performance of that investment compared to the market.  correct  false